

Trigonometry Improv'd,
Samuel AND *Sumner*

Projection of the SPHERE,
Made Easy.

TEACHING

The Projection of the Sphere Ortho-
graphick, and Stereographick : As al-
so, Trigonometry Plain and Spherical ;
with plain and intelligible Reasons for
the various and most useful Methods,
both in Projection and Calculation ;
with the Application of the whole to
Astronomy, Dialling, and Geography.

By HENRY WILSON.

L O N D O N :

Printed by H. P. for J. SENEX, at the *Globe*
in *Salisbury-Court* ; and W. TAYLOR, at the
Ship in *Pater-Noster-Row*. MDCCXX.





To the Worshipful the Masters, Wardens, and Assistants of the *Trinity-house* of *Deptford-Strand*.

I*T may be thought Surprising, if not accounted great Presumption, to Dedicate so small a Treatise to so honourable and useful a Corporation. But my Hopes that this plain, tho' concise Piece of Trigonometry, well applied, may at least contribute to the qualifying the Studios for some of those Publick Mathematical Services, to*
A 2 *which*

THE DEDICATION.

which they cannot be introduced without your Examination and Approbation, hath encouraged me to cast this my Mite into your Treasury, which that your Worships may please to accept of, is the humble Desire of

Your most

humble Servant,

HENRY WILSON.

TO.



TO THE READER.

ALtho' the great Number of Books of all sorts now extant in the World, might be a sufficient Discouragement to any Author from offering any Thing more to the Press; yet the Consideration of that common Complaint of a great Part of those who have been at much Cost and Pains to learn, viz. *that they forget a considerable Part of what they have learned, before they have the Opportunity to put it in Practice*, hath animated me to venture into the World with this my Manual of *Trigonometry*, in which I have endeavoured to serve Learners of all sorts; the Substance of Projection of the Sphere and *Trigonometry*, being compriz'd in such a moderate Volume, that it cannot be thought (by those that have any Opportunity and Desire to learn) a Difficulty to procure Money to buy it, or Time to read it; the Rules so clear, that it is not hard to understand it; and the Reasons for them so plain, that it will be very easy to retain and remember it, and apply it to Practice as Occasion requires.

To the READER.

I have first begun with the Rudiments of Geometry, so far as is necessary to the Performance of what follows.

2. Plain Trigonometry Right-angled and Oblique, in order to the right understanding of which, I have inserted some necessary Definitions and Axioms, with the Construction of Sines, Tangents, and Secants so explain'd, that the Learner may see the Reason why the Sides of the Triangle become Sines, Tangents, or Secants, to their respective Angles, which Reasons once learn'd, the Rules cannot be easily forgot.

3. Before I have entred upon Spherical Trigonometry, I have handled the Stereographick Projection of the Sphere, with general Rules for doing it upon the Plain of any great Circle. I have Instanced in five of the Principal, viz. The Meridian, Horizon, Equator, Ecliptick, and Prime Vertical; which, I suppose, with the Reasons there given for the several Appearances of the Projection, according to the various Positions of the Eye, will be sufficient for the Information of the Learner in all the Varieties of that Projection.

I confess I have in this Particular deviated from the Order of some Authors, viz. either those who treat of Spherical Trigonometry and Astronomy, without speaking any Thing of the Projection of the Sphere, or those, who in the Order of their Books speak of Spherical Trigonometry first, and Projection of the Sphere afterwards. I cannot but think it more proper and instructive to the Learner to be first taught the Projection of the Sphere, and
the

To the R E A D E R.

the several Circles which constitute those Triangles, which are the Subject of Spherical Trigonometry, before they fall upon the Calculation; for it seems evidently more easy and natural to form a Proportion from what is given for the Solution of any proposed Triangle, after it is projected in its proper Appearance, than to perform the Calculation first and the Projection afterward, the same *Data* that serves for one, serving also for the other, and the Proportion for Solution sometimes depending upon the Proportions of the Sines of Sides, to the Sines of their opposite Angles, sometimes requiring a Perpendicular to be let fall, &c. which Varieties may be best conceived after the Triangle is projected, and that has induced me to assume this Method.

In all my Problems in Astronomy, I have always chused to Project upon the Plain of such a great Circle, as may be most Commodious for Performance, and may come under the general Rule given (*pag.* 119. if in Oblique) whereas to those that learn to Project only upon the Plain of the Meridian, the Projection from some *Data* is rendred impossible.

And because nothing should be wanting to make Spherical Operations intelligible, I have not only explained my Lord *Nepain's* Proportion, but also that which I have called the *Globical Projection of the Sphere*, and therein deduced Proportions for the solution of all the Varieties of Right-angled Spherical Triangles (and consequently of Oblique-angled Triangles reduced to Right-angled by Perpendiculars) from no other Principle than the simple Construction
of

To the READER.

of Sines (being Perpendiculars let fall) and Tangents (being Perpendiculars raised from the End of Radius.) And for the better illustration of both, I have made use of this in the Cases of Trigonometry, and of my Lord Ne-pain's Proportion in Astronomy, and have also added the Orthographick Projection of the Sphere, and the Solution of the most necessary Questions in Astronomy thereupon, which is performed by Scale and Compasses only, viz. by the Chords and Sines on the Gunter's Scale; and I have also inserted the most material and Fundamental Problems in Dialling and Geography, and concluded with some useful Tables to find the Rising, Setting, &c. of the Sun, Moon, or Planets.

Altho' I have not been so large in the Application of Trigonometry to Dialling and Geography, as a larger Treatise would admit of; yet I have given such demonstrable Reasons for the most material Problems therein, as is sufficient to incite the Learner to, and inform him in those useful Parts of the Mathematicks; which in this, as in others of my publick Performances, is the chief Design of

A true Lover of the Mathematicks,

HENRY WILSON.

N. B. *I have not used any Symbols or Contractions, except that one in Spherical Projections [P C Q] which signifies, Draw the Primitive Circle and Quarter it, that is, Cross it at Right-Angles with two Diameters.*

The



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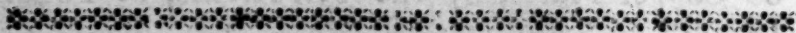


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N. B. The Author *Henry Wilson*, Surveys Land, draws Maps of Estates, &c. and is willing to serve any Gentleman in that, or other Mathematical Performances, if they please to enquire for him at the Place above-mentioned.





CHAP. I.

SECT. I.

GEOMETRICAL PROBLEMS.

PROBLEM I.

*From a given Point A in the Middle of a Line,
to Erect a Perpendicular.*



WITH any Extent in your Com-
passes, and one Foot in *A*, make
the Marks *c* and *d*; then with
any greater Extent, and
one Foot in *c*, make the Arch *e*, and with the

Plate I.

Fig. I.

same Extent, and one Foot in *d*, cross
the said Arch in *e*; a Line drawn from the
Intersection of the Arches in *e* to the given
Point *A*, is the Perpendicular required.

B

PRO-

PROBLEM II.

From a Point given without, to let fall a Perpendicular to a Line given.

With any Extent more than the nearest Distance between the Point and the given Line, and one Foot in the given Point X , cross the Line in m and n ; then with the same
 Plate I. Extent, (or any other greater than
 Fig. 2. half the Distance $m n$) placing one Foot in m and n , make two Arches to cross each other in r , lay a Ruler from x to the Crossing at r , and by it draw $x b$, it is the Perpendicular required.

PROBLEM III.

From the End of a given Line, to raise a Perpendicular.

With any Extent, and one Foot in the End of the Line at B , place the other Foot any where above the Line, as at C , and keeping that Point at C , turn the other about, and cross the Line at D , and turning your Com-
 Plate I. passes, make the Arch E ; lay a Ruler
 Fig. 3. from the Crossing at D over the Point C , and cross the Arch E , a Line drawn from the Crossing at E to the Point B , is the Perpendicular required.

PROBLEM IV.

From a Point given over the End of a Line, to let fall a Perpendicular.

From the given Point B draw the Line $B D$, to cut the Line $D E$ at any convenient Distance
 from

from the End E , whereabout the Perpendicular is to fall, divide the said Line BD into two equal Parts at F , and placing one Foot in F , with the Extent FB , or FD , cross the Line DE in E , the Line BE being drawn, is the Perpendicular required. Plate 1. Fig. 4.

PROBLEM V.

To divide a given Right Line into two Equal Parts, properly called Bissecting a Line.

Let the Line be AC , with any Extent greater than half the Line, and one Foot in A make the Arch bd ; then with one Foot in C cross the said Arch in b and d ; a Line drawn from b to d , will bissect the Line AC in e , as was required. Plate 1. Fig. 5.

PROBLEM VI.

At a given Distance, to draw a Line Parallel to a given Line.

Take the given Distance in your Compasses, and with one Foot any where towards each End of the given Line, as in a and b , sweep the small Arches c and d ; a Line drawn so as just to touch the Extremity of those Arches, shall be the Parallel required. Plate 1. Fig. 6.

PROBLEM VII.

To bring any three Points into the Circumference of a Circle, provided they are not situate in a Right Line.

Let the Points be A, B and C , first with any Extent more than half AB , and one Foot in A ,
B 2
make

make the Arch $d m e$, and with the same Extent, and one Foot in B describe the Arch $d n e$, to cross the former in d and e ; then
 Plate 1. with any Extent more than half BC ,
 Fig. 7. and one Foot in B and C , describe the Arches $h i g$, and $h r g$, crossing each other in h and g , then through the two Crossings h and g , draw the Line $h g o$; likewise through d and e draw $d e o$, and where these Lines cross each other, (as in o) is the Center of the Circle, and the Distance $o A$, $o B$, or $o C$, will sweep the Circle required.

Note, By the same Means may be found the Center of any Circle or Arch of a Circle by assuming any three Points, and finding the Center as above.

PROBLEM VIII.

To lay down any given Angle at the Point B in the Line BC , suppose of 40 Degrees.

With the Chord of 60, (always) and one Foot in the given Point B , describe the Arch $e g$; then take your given Degrees
 Plate 1. 40 off the same Chord, and set from
 Fig. 8. g to e , and from B through e draw $B D$, it will make an Angle of 40 Degrees at B , with the Line BC .

S E C T. II.

DEFINITIONS and AXIOMS.

EVERY Triangle consists of three Sides, and three Angles.

In a right angled Triangle, the longest or slope Side is called the *Hipotenuse*; the other two Sides including the right Angle, are called *Legs* or *Sides*; or sometimes one is called the *Base*, and the other the *Perpendicular*, and they may be called so alternately according to the Position of the Triangle.

An Angle is commonly express'd either by a single Letter, as the Angle *B*, or by three Letters, as the Angle *ABC*, whereof the Middle signifies the Angle.

An Acute Angle is less than 90 Degrees, as *ACB*.

A Right Angle is 90 Degrees, as *ABC*.

An Obtuse Angle is more than 90 Plate 1.
Degrees, as the Angle *ACG*. Fig. 13.

The three Angles of any right lined Triangle added together make 180 Degrees; therefore the two Acute Angles of any right angled Triangle make always 90 Degrees, because the right Angle is 90, therefore one Acute Angle of a right angled Triangle, or two Angles of an Oblique Triangle, being known, the other may be easily found.

When the Sine, Tangent, &c. of an Obtuse Angle is required, you must instead of that
B 3 take

6 Construction of Sines, &c.

take its Complement to 180; thus the Sine of 130 Degrees is the Sine of 50 Degrees, &c.

Co Ar, or Complement Arithmetical, is found by subtracting every Figure in a Sine or Tangent from 9, except that next the Right Hand, and that is to be subtracted from 10. Thus the *Co Ar* of the Sine of 29^d, 6^m, is 0, 31306, but may be more readily found by Inspection in the Table of Logarithmal Sines, Tangents, and Secants, thus:

For <i>Co</i>	{	Sine	}	Take	{	Sec. Compl.	}	reject	
<i>Ar</i> of		Tang.				Tang. Compl.			Ra-
		Secant				Sine Compl.			

And you have the Complement Arithmetical required.

The chief Use of *Co Ar* is, when *Radius* is not one of the given Terms; for then by taking the *Co Ar* of the first Term instead of the Sine or Tangent itself, and adding it to the other two Terms, the Sum abating *Radius*, is the fourth Term required.

A Definition of the Sines, Tangents, &c.

In order to the right Understanding of Trigonometry, it is necessary to know
Plate I. what is a Sine, Tangent, Secant, &c.
Fig. 10. of which take the following Definition.

Every great Circle is supposed to contain 360 Degrees, the half Circle 180, and the Quadrant (upon which the Sines, Tangents, &c. are projected) 90 Degrees.

The *Radius*, Root, or Foundation of the Projection, is the Extent with which the Arch is swept,

swept, or the Semidiameter of the Circle, and is commonly supposed to be divided into 10000 Parts, as be .

A right Sine is a right Line drawn from the given Degree, till it fall at right Angles upon that Semidiameter, from whose Extremity it is number'd as g , f is the Sine of 45 Degrees, because it is a Perpendicular, let fall upon the Radius eb , and the 45 Degrees is number'd from b .

The Complement of an Arch is so much, as it is less than 90, and the Sine Complements, Tangent Complements, &c. are only the Sines, Tangents, &c. of those Differences; thus gi equal to fe , is the Sine Complement of 45, and is the Sine also of 45 Degrees, if number'd back from 90, because it is a Perpendicular let fall upon the Line eb , from whose Extremity (b) the 45 Degrees are number'd to g .

A Tangent Line is the Line bd continued, being Perpendicular to the Radius eb , and just touching the Circle and the Tangent of any Arch is number'd upon that Line, and limited by a right Line drawn from the Center through the Degree proposed, and continued till it cut the Tangent Line; thus bd is the Tangent of 45 Degrees.

A Secant is the Line so drawn from the Center, till it cut the Tangent Line; thus ed is the Secant of 45 Degrees.

A Versed Sine is that Part of the Diameter comprehended between the Circle and the End of the Right Sine, as fb is the Versed Sine of 45 Degrees.

A Chord is a right Line drawn between the Extremities of the given Arch, as the Line bg is the Chord of 45 Degrees. Note, The Chord of any Arch is equal to twice the Sine of half the said Arch.

From this Diagram well understood and apply'd, are deduced Rules and Proportions for solving the following Cases of Trigonometry, observing, that as *Radius* is the Semidiameter of the Circle, so in the Application, whatsoever Side of a right angled Triangle you would make *Radius*, take that Side in your Compasses, and with one Foot in the Angle at the Base sweep a Circle, then will the Angle at the Base become the Angle at the Center, and each Side will at once discover whether it be Sine Tangent, or Secant, according to the Description above.

Suppose in the Triangle abc , you would make the Base ab *Radius*, take ab in
 Plate 1. your Compasses, and with one Foot
 Fig. 9. in a draw the Arch bd , and draw ad Perpendicular to ab , then is abd a Quadrant, and by the foregoing Definitions bc is the Tangent, and ac the Secant of the Angle bac .

But supposing the Triangle efg to be the same as abc , only you would make
 Plate 1. the Hipotenuse eg , *Radius*, then gf
 Fig. 10. is become the Sine, and fe (equal to gi) the Sine Complement of the Angle e , and these will be the Terms necessary to be made use of in the Solution.

All

Plain Trigonometry.

All these Things rightly understood, all the Varieties in right lined Triangles may be reduced to these four *Axioms*.

A X I O M I.

In all right angled plain Triangles, if one Side be made *Radius*, the other Sides will be Sines, Tangents, or Secants. For first,

If a Leg be made *Radius*, the other Leg will be the Tangent of the Angle at the Base, and the Hipotenuse the Secant of the same Angle.

Plate 1. Fig. 9.

But if the Hipotenuse be made *Radius*, the Leg opposite to the Angle at the Base is the Sine of the said Angle, and the other its Sine Complement. Plate 1. Fig. 10.

A X I O M II.

In all plain Triangles the Sides are proportionable to the Sines of their opposite Angles.

In right angled Triangles it is evident, if you make the Hipotenuse *Radius*, that the Legs are the projected Sines of their opposite Angles, and consequently bear the same Proportion to the Hipotenuse, and between themselves, that the Sines in the Table bear to *Radius*, and to one another. (See Plate 1. Fig. 10.)

A X I O M III.

In all Triangles, as the Sum of the Legs including any Angle is to their Difference, so is the Tangent of half the Sum of the unknown Angles, to the Tangent of half their Difference.

A X I O M IV.

In all right lined Triangles; as the Base (or longest Side) to the Sum of the other two Sides, so is the Difference of the said shortest Sides to the Difference of the Segments of the Base; see this explain'd in Case the Sixth of Oblique Triangles.

SECT. III.

PLAIN TRIGONOMETRY Right-angled.

CASE I.

THE Hipotenuse and the Angles being given, to find a Leg.

Given { Hipotenuse AC 86 } Required
 { The Angle BAC $36^{\circ} 30'$ } CB .

CONSTRUCTION.

Draw AB at Pleasure, and by *Prob. 8.* make the Angle A $36^{\circ} 30'$, upon AC set off 86 from A to C , and by *Prob. 4.* let fall the Perpendicular CB , then shall AB (measured on the same Scale of equal Parts, that AC was taken from) be 51. Plate 1. Fig. 11.

CALCULATION.

First making AC Radius, (which is most easy, because it comes under the Proportion of Sides to the Sines of their opposite Angles) then (by *Plate 1. Fig. 10.*) CB is the Sine of the Angle A , therefore

As Radius	—	10. 00000
To AC	86	1. 93449
So Sine of the Angle A $36^{\circ} 30'$		9. 77438

To Side opposite, BC 51	=	1. 70887
---------------------------	---	----------

But

But making AB Radius, then AC is the Secant, and BC the Tangent of the Angle A , (by Plate 1. Fig. 9.) therefore,

As Secant of the Angle A $36\ 30$ - Co Ar 0.90517

To AC ————— 86 — 1.93449

So Tangent of the Angle A $36\ 30$ - 9.86920

To Side BC ————— 51 — 1.70886

But making BC Radius, AC will (by Plate 1. Fig. 9.) be the Secant of the Angle C , (which is the Complement of A , viz. $53\ 30$) and then

As Secant of the Angle C $53\ 30$ - 10.22561

To AC ————— 86 — 1.93449

So Radius ————— 10.00000

To Side BC ————— 51 — 1.70888

I shall not trouble the Reader with all these Varieties in the rest, supposing this with a little Application sufficient to give the Learner an Insight into the making every Side Radius; and the Reason of the Proportions arising from thence.

CASE II.

Given the Angles and a Leg, to find the Hypotenuse.

Given { The Angle A 40.02 Required:
 { The Leg AB 500 } AC .

CONSTRU-

CONSTRUCTION.

Make AB 500, and set off the given Angle, as in Case 1. at A , and draw AC at Pleasure; then from B (by Prob. 3.) raise the Perpendicular BC to cut AC in C , and the Triangle is finished.

Plate I.
Fig. 11.

CALCULATION.

Making AC Radius, AB will be the Sine of the Angle at C , then

As Sine of the Angle C	— 500 —	9.88425
To Side AB	— 500 —	2.69897
So Radius	— —	10.00000

To Side AC required — 653 — 2,81472

CASE III.

Given the Angles and a Leg, to find the other Leg.

Given	{	The Angle at A 40°	}	Required
		The Leg AB -- 500		

CONSTRUCTION.

As in Case the second, the same Thing being given.

CALCULATION.

CALCULATION.

Making AB Radius, BC will be the Tangent of the Angle at A therefore,

As Radius	—	—	10.00000
To AB 500	—	—	2.69897
So Tangent of the Angle A 40°	—	—	9.92381
To BC required	—	419	— 2.62278

CASE IV.

Given the Hipotenuse and a Leg, to find an Angle.

Given { The Hipotenuse AC 720 } Require the
 { The Leg AB — 550 } Angle A .

CONSTRUCTION.

Make AB 550, and at B erect the Perpendicular BC at Pleasure, by *Prob. 3.*
Plate I. then with the Extent AC 720, and
Fig. 11. placing one Foot in A , cross the Perpendicular BC in C , and draw AC , and it is done.

CALCULATION.

Making AC Radius, AB will be the Sine of the Angle C , or Sine Complement of the Angle A , required therefore

As AC	—	720	—	2.85733
To Radius				10.00000
So AB	—	550	—	2.74036

To Sine Comp. of Angle A , 40° 11' — 9.88303

CASE

CASE V.

Given the Hipotenuse and a Leg, to find the other Leg.

Given { The Hipotenuse AC 720 } requir'd the
 { The Leg AB — 550 } Leg BC .

CONSTRUCTION.

As in Case 4. being the same Data.

CALCULATION.

This requires two Operations; first by Case the 4th, to find the Angle at A , which is already found to be 40.11 ; then if you make AB Radius, BC is the Tangent of the Angle at A , but AC being made Radius, BC is the Sine of the said Angle, and according to the latter.

As Radius	—	10.00000
To AC	720 —	2.85733
So Sine of the Angle at A	40.11 —	9.80971
To the Leg BC required	464 —	2.66704

CASE VI.

The Legs given, to find an Angle.

Given { The Leg AB 360 } Required the An-
 { The Leg BC 430 } gle A .

plate 1.

Fig. 12.

CONSTRU-

CONSTRUCTION.

Make AB 360, and at B by *prob. 3.* erect the Perpendicular BC , upon which set off 430, from B to C , and draw AC , and 'tis done.

CALCULATION.

Making AB Radius, BC will be the Tangent of the Angle at A , therefore

As AB	—	360	—	2. 55630
To Radius	—		—	10. 00000
So BC	—	430	—	2. 63346

To Tang. of the Angle A 5.04 — 10, 07716

CASE VII.

The Legs given, to find the Hipotenuse.

Given $\begin{cases} AB & 360 \\ BC & 430 \end{cases}$ Required AC .

Construction as Case the sixth.

CALCULATION.

Find the Angle A by Case the sixth, which is $50^{\circ} 4'$ then

Making AC Radius, BC will be the Sine of the Angle A , therefore

As Sine of the Angle A , $50^{\circ} 4'$	—		—	2. 88467
To Side BC	—	430	—	2. 63346
So Radius	—		—	10. 00000

To AC required — 560. 8 — 2. 74879

There

There is another Method when the Legs are given, to find the Hipotenuse and the Angles, viz. Multiply each Leg by itself, and add the two Products together, extract the square Root of that Sum, it shall be the Hipotenuse required, (*Euclid. Liv. I. Prop. 47.*) and having found the Hipotenuse, take half the longer Leg, and add it to the Hipotenuse, and say, As that Sum is to 86, so is the shorter Leg to its opposite Angle; and having found one of the acute Angles, the other is easily found, it being its Complement to 90 Degrees.

EXAMPLE.

In the foregoing Triangle in Case 6 and 7, AB 360, BC 430. First to find AC .

			314500 (560	
			25	
			—	
360	430	106)	645	Making the Re-
360	430		636	mainder 900. a
—	—		—	Numerator to
216	129	112)	900	the Root. Dou-
108	172			bled the Hi-
—	—			potenuse is 560-
129600	184900			$1\frac{200}{1120}$ which re-
	129600			duced is 560-
	—			$\frac{45}{8}$, which in
Sum of	314500			Decimals is 560,
the Pr od.				8 nearest

Then

Then for the Angle C opposite to the lesser Side,

AC (to avoid Fractions) is 561

The longer Leg BC 430, its half is 215

776

As 776 86 360

86

216

288

776)30960(39 $\frac{22}{776}$

2328

7680

6984

1 696 1

For the lesser Angle, viz. C , whose Complement 50 $\frac{22}{776}$, answers sufficiently near the Truth for common Use.

To illustrate this Method of finding an Angle.

CONSTRUCTION.

Retaining still the former Triangle ABC .

Plate I.

Fig. 13.

Continue the longer Leg CB both Ways, and with the Extent AC , and one Foot in the least Angle) describe the Circle FAD , make FG equal to FC , erect the Perpendicular or Tangent Line DE , and draw GA continued, till it cut the said Tangent DE in E .

This

This Proportion is grounded upon a Supposition, that the Arch AD differs insensibly from the Tangent, Line DE ; and indeed the less the Angle at the Center, they still differ the less: And even in this Case, where the Angle at C is near 40 Degrees, the Error is not material. Now, if we grant AD equal to ED , and the Diameter of a Circle to be to the Circumference, as 113 to 355, the Proportion follows thus.

In the Similar Triangles GBA , and GDE , it is as GB to BA , so GD to DE , (*Euclid. Lib. VI. Prop. 4.*) and if DA be equal to DE , it is as GB to BA , so GD to DA ; and because we are endeavouring to measure the Angle BCA by the Arch AD , it must be supposed to be an Arch of a Circle, the Whole of which contains 360 Parts or Degrees, and its half DAF 180, and by the above-mentioned Proportion its Diameter is $114\frac{2}{3}$, such Parts whereof the whole Circumference is 360, the half of that, viz. $57\frac{1}{3}$ is Radius, and 3 times $57\frac{1}{3}$ is $171\frac{2}{3}$. Now, because by Construction GD is 3 times Radius, viz. $171\frac{2}{3}$, which for Evenness of Numbers we will call 172, and GF is equal to FC , equal to CA the Hipotenuse and CB is the longer Leg, it will be as GB (twice the Hipotenuse, and once the longer Leg) to GD (thrice Radius, or) 172, so BA the shorter Leg, to DE , which is granted equal to DA , which (DA) is the Measure of the Angle E required.

And by taking half the two first Terms, it will be as the Hipotenuse added to half the longer Leg is to 86, (the $\frac{1}{2}$ of 172) so the shorter Leg is to its opposite Angle.

S E C T. IV.

Of Oblique Plain TRIANGLES.

C A S E I.

Two Angles and a Side opposite to one of them being given, to find the Side opposite to the other.

Given $\left\{ \begin{array}{l} \text{The Angle } D \ 35 \\ \text{The Angle } E \ 40 \\ \text{The Side } EF \ 380 \end{array} \right\}$ Required the Side DE .

Having given the Angles D and E , you have consequently the Angle F , being the Complement of the other to 180 .

The Angle D	$35 \ 0$	From	$180 \ 0$
The Angle E	$40 \ 0$	Subtract	$75 \ 0$
Sum	$75 \ 0$	Rest the Angle F	$105 \ 0$

C O N S T R U C T I O N.

Draw DE at Pleasure, and by *Prob. 8.* make the Angle E 40 Degrees; upon the plate 1. Side EF , set off 380 from E to F , *Fig. 14.* and at F make an Angle of 105 Degrees drawing FD ; and the Triangle is finished.

ALUCLA

CALCULATION.

This is solved by *Axiom* the Second, and the Proportion is,

As Sine of the Angle D 35° Co.Ar-0.24140
 To Side opposite FE 380 --- 2.57978
 So Sine of the Angle E 40° --- 9.80806
 To Side opposite FD required 426 --- 2.62924

CASE II.

Two Sides and an Angle opposite to one of them, being given, to find the Angle opposite to the other.

Given { The Side DE 500
 The Side EF 420
 The Angle D 30° } required the Angle F .

This Case is ambiguous, and may have the Angle F , either acute or obtuse, which therefore must be known before the Quantity of the Angle can be determined.

CON-

CONSTRUCTION.

Draw FD at Pleasure, and by *Prob. 8.* make at D the given Angle 30 , and draw DE 500 ; then with the Side EF 420 , in your *Plate 1.* Compasses, and one Foot in E , cross *Fig. 15.* the Line FD , in F and F , one making the Angle EDF acute, the other obtuse: But if it be first known whether the Angle F be acute or obtuse, you need draw but one of the Lines EF , according as the Angle is, and the Triangle is finished.

CALCULATION.

By *Axiom the Second*, the Proportion is,

As Side FE	—	400	---	Co. Ar.	7.37676
To Sine of the Angle opposite D	} $30^\circ 0'$	—	---		9.69897
So Side ED					

To Sine of the opposite Angle F	} $36^\circ 32'$ if acute, 7 $2143^\circ 28'$ if obtuse	---		9.77470

CASE III.

Two Sides and an Angle opposite to one of them being given, to find the third Side.

Given	{	The Side DE	550	} required the Side FD .
		The Side EF	420	
		The Angle D	$30^\circ 0'$	

CON-

CONSTRUCTION.

As in *Case* the Second.

CALCULATION.

Having found the Angle *F*, as in *Case* the Second, the Angle *E* is found by *Subtraction*; if *F* be acute, the Angle *E* is 113 28; if *F* be obtuse, the Angle at *E* is 6 32; then first if the Angle *F* be acute.

As Sine of the Angle <i>F</i> -36 32--Co. Ar-	0.22527
To Side opposite <i>ED</i> -- 500 --	2.69897
To Sine of the Angle <i>E</i> -113 28 --	9.96250
To Side opposite <i>FD</i> -- 770 --	<u>2.88674</u>

But if the Angle *F* be obtuse,

As the Sine of the Angle <i>F</i> -143 28-Co. ar-	0.22527
To Side opposite <i>ED</i> -- 500 --	2.69897
So Sine of the Angle <i>E</i> -- 6 32 --	<u>9.05607</u>
To Side opposite <i>FD</i> required 95 ---	1.98031

CASE IV.

Two Sides and their contained Angle given, to find either of the other Angles (consequently both.)

Given

Plain Trigonometry.

Given $\left\{ \begin{array}{l} \text{The Side } FE \quad 420 \\ \text{The Side } FD \quad 770 \\ \text{The Angle } F \quad 36,32 \end{array} \right\}$ To find the Angle E .

CONSTRUCTION.

Make FD 770 and by *Prob. 8.* make the Angle F 36,32, and draw FE , upon which set off 420 from F to E , and draw the Line ED , and 'tis done.

CALCULATION.

Plate I. The Performance of this depends
Fig. 16. upon *Axiom* the Third, viz.

As Sum of the given Sides 1190-Co.Ar-6.92446

To the Difference of
the given Sides $\left\{ \begin{array}{l} 350 \quad -- \quad 2.54406 \end{array} \right.$

So Tang. $\frac{1}{2}$ Sum of
unknown Angles $\left\{ \begin{array}{l} 71 \quad 44 \quad -- \quad 10.48138 \end{array} \right.$

To Tang. $\frac{1}{2}$ their Difference 41 43 $\frac{1}{2}$ 9.94990

Half Difference added to $\left\{ \begin{array}{l} 113 \quad 27 \end{array} \right.$ the Angle E .

Subtracted from the
half Sum $\left\{ \begin{array}{l} 30 \quad 1 \end{array} \right.$ the Angle D .

CASE V.

Given two Sides, and their contained Angle,
to find a third Side.

Given

Given $\left\{ \begin{array}{l} \text{The Side } FD \ 770 \\ \text{The Side } FE \ 420 \\ \text{The Angle } F \ 36.32 \end{array} \right\}$ required the Side ED .

CONSTRUCTION.

The same *Data* and Construction as the foregoing Case.

CALCULATION.

Find the Angle as in *Case* the 4th, where the Angle D is $30^{\circ} \ 1'$ then, as in *Case* the first.

As Sine of the Angle D --- 30.1 --- Co. ar. --- 0.30081
 To Side opposite EF --- 420 --- 2.62324
 So Sine of the Angle F --- $36 \ 32$ --- 9.77472
 To Side opposite ED required 500 --- 2.69877

CASE VI.

Three Sides given to find an Angle.

Given $\left\{ \begin{array}{l} \text{The Side } FD \ 770 \\ \text{The Side } FE \ 420 \\ \text{The Side } ED \ 500 \end{array} \right\}$ required the Angle F .

C

CON-

CONSTRUCTION.

Plate 1. Make FD 770, then from the same *Fig. 17.* Scale take in your Compasses 420, and with one Foot in F , make the Arch E ; then with 500 in your Compasses, and one Foot in D , cross the Arch E in E , draw FE and DE , and 'tis done.

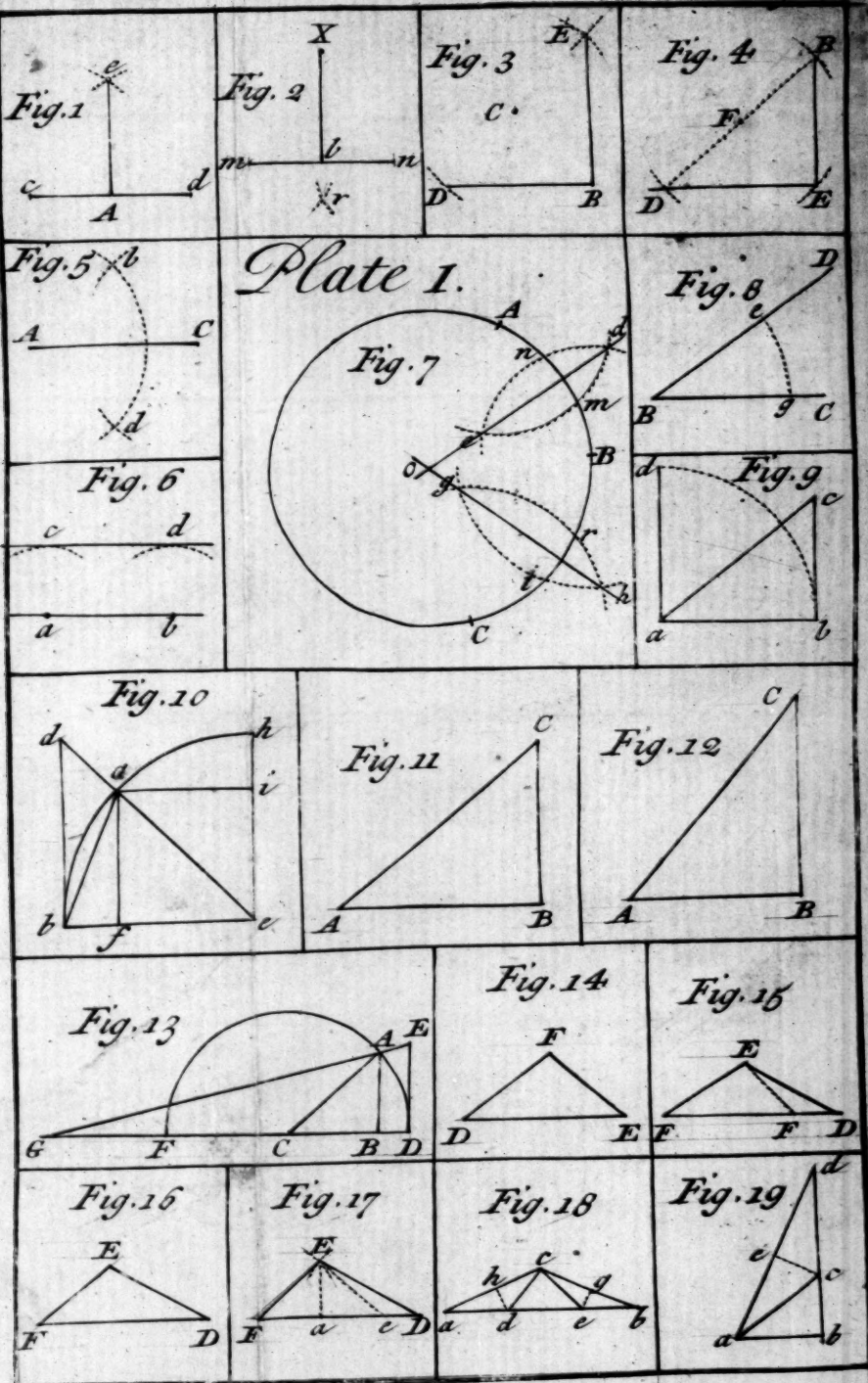
CALCULATION.

The Solution of this depends upon the 4th *Axiom*; for before you can proceed to the Operation, the most common and easy Way, you must first suppose the oblique Triangle to be divided into two right angled ones, by the Perpendicular let fall from E to a ; and to find the Segments Fa , and aD , the Rule by *Axiom* 4, is,

As the Base	—	—	770-Co. ar-7.11351
To the Sum of the Sides	920	---	2.96378
So the Difference of the Sides	80	---	1.90309
To the Difference of the Bases	95	---	1.98038

The Difference De 95, taken from the whole Base FD 770; half the Remainder, viz. Fa 337.5 is the Base of the right angled Triangle FaE , in which to find the Angle F , making FE Radius, Fa will be the Sine Complement of the Angle E ; therefore,

As



As FE	---	----	420	Co. ar-7.37676
To Radius	---	----	---	10.00000
So is Fa	---	----	337.5	--- 2.52827
To S. C. of the Angle	{ 36 32			
F required	--- 9.90503			

Three Sides of any right lin'd Triangle, in one Sum given, and the Angles also given to lay it down, and find the Sides severally.

{ Sum of the Sides 80 }
 Given { The Angles { 55 ° } to find a Side.
 { 46 ° }
 { 79 ° }

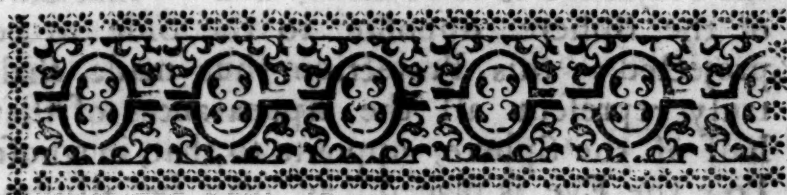
Draw ab 80. Make the Angle b Plate 1. ac equal to half one of the given Fig. 18. Angles; as suppose, half of 55.0, viz. 27 30, and draw ac , then make the Angle abc half 46 °, viz. 23 °, and draw bc to cut ac in c , from the Middle of the Lines ac and cb draw the Perpendiculars bd and ge , and where they cut the Line ab , as in d and e , make Marks, to which draw the Lines cd and ce , then cde is the Triangle required.

Then in the Triangle acb you have given the Angle acb 129 30 (the Angle, acd being half the Angle cde Eucl. Lib. 1. Prop. 32. and ecb is half ced by the same,) and there is also given the Side ab 80, and the Angle b 23:0 to find the Side ac , which being found, you have

given in the Triangle $a c d$ the Angle $e d a$ $125^{\circ} 0'$, it being the Complement of $c d e$ to 180° , *Eucl. Lib. 1. Prop. 13.* and the Side opposite $a c$, and the Angle $c a d$ $27^{\circ} 30'$ to find its opposite Side $c d$; and when that is found, you have in the Triangle $d c e$, one Side, and all the Angles to find the other Sides severally.

In the right angled Triangle $a b c$, there is given $a b$ 96 , and $b c$ and $c a$ in one Sum 220 , to find the Sides and Angles severally.

Draw $b a$ 96 , and perpendicular *Plate 1.* thereto at b , draw $b d$ 220 , and draw *Fig. 19.* $a d$, which bissect in e , and draw the Perpendicular ec , to cut $b d$ in c , and draw $a c$, then is $a b c$ the Triangle required. Then for Calculation in the Triangle $a b d$, you have given $b d$ 220 , and $a b$ 96 , to find the Angle at d , which is equal to the Angle $c a e$ (because their opposite Sides are equal.) Likewise the Angle $d a b$ is the Complement of $a d b$ to 90° ; then from the Angle $d a b$ subtract the Angle $e a c$, there remains the Angle $c a b$, whose Complement is the Angle $a c b$, then in the said Triangle $a b c$, there is given the Base $a b$, and the Angles a and b , to find the Hipotenuse by *Case 2.* or the Perpendicular by *Case 3.* but I shall leave the Operation to the Learner's Practice.



CHAP. II.

SECT. I.

Of the PROJECTION of the SPHERE.



HERE are several Ways of Projecting the Sphere, viz. Stereographick, Orthographick, and Gnomonickal. I shall only speak of the Stereographick at this time, as

being most useful in Trigonometrical Operations, and shall refer the other to their proper Places afterwards.

Stereographick Projection of the Sphere, is the Projection of the Sphere or Globe, upon a Plain, passing through the Center thereof; the Eye being supposed to be placed upon the Superficies or Surface of the Globe, perpendicular to the Center of the said Plain, or in the Pole of that great Circle, upon which

the Projection is to be made, and on that Side of the Sphere opposite to the Side to be projected; this Projection thus made, shall truly represent all the Appearances of that half of the Sphere, opposite to that, in whose Pole the Eye is placed, as may be illustrated by the following Similitude.

Suppose a Sphere or Globe were made of Glass, or any transparent Metal, on which were described all the Points and Circles of the Sphere (as *Zenith*, *Nadir*, *Equator*, *Ecliptick*, &c.) in black Lines that might be discerned through the Globe; and admit there were a Glass Plain passing through the Center of the Globe, cutting it in two equal Parts in that great Circle, upon whose Plain the Projection is to be made; as for Instance, suppose I were to project the Western Hemisphere upon the Plain of the Meridian, having a transparent Globe, prepared as before described, and the transparent Plain to pass through the Center of the Sphere, and cut it in two Parts in the Meridian (the great Circle upon which the Projection is to be made) if the Eye be supposed to be placed in the East Point of the Horizon, upon the Surface of the Sphere; and observe, where the Meridians, Azimuths, &c. on the West Side of the Sphere cuts the Plain passing through the Middle of the Sphere: These Circles upon that Plain shall truly represent the Projection of the Western Hemisphere upon the Plain of the Meridian; for the Projection of the Sphere, of what kind soever, is no more but the

the Representation of those Circles and Points upon a Plain, which are imagined to be described in the Sphere, or upon the Celestial or Terrestrial Globe.

The imaginary Points, are 1. The North and South Poles of the World, or of the Equator. 2. The Poles of the Ecliptick. 3. The Equinoctial Points *Aries* and *Libra*, where the Ecliptick cuts the Equator. 4. The *Zenith* and *Nadir*.

The Circles of the Sphere, are either greater or lesser Circles. The greater Circles are those that divide the Sphere itself in two equal Parts; as 1. The Horizon which divides that Part of the Sphere visible to us, from that Part that is invisible. 2. The Meridian, which divides the East half from the West. 3. The Equator, which divides the North Half from the South. 4. The Ecliptick. 5. The Prime Vertical, &c. of all which see a more particular Description afterwards.

The lesser Circles divide the Sphere in two unequal Parts, as the Almicanthar or Parallels of Altitude, and also the Parallels of Declination, so called, because they are Parallel to the Equator, &c.

The Sphere may be Stereographically projected upon the Plain of any great Circle; I have for the Learner's Information, instanced in five Varieties, having projected the Sphere.

Upon the Plain
of the

{	Meridian, Fig. 1.	}	Plate 5.
	Horizon, Fig. 2.		
	Equator, Fig. 3.		
	Ecliptick, Fig. 4.		
	Prime Vertical, Fig. 5.		

Upon the Plain of whatsoever great Circle the Sphere be projected, all the Circles in the Sphere will be Circles in the Plain of the Projection, and if continued to perfect Circles, will answer all the Representations and Conditions of Measuring, &c. without the primitive Circle, as well as within it; and also the Angles made by the Interfection of two great Circles on the Surface of the Sphere, are equal to the Angles made by their Representatives on the Plain of the Projection.

In any Projection, that Circle upon whose Plain the Sphere is projected, is called the primitive Circle, and is always a perfect Circle described about the whole Projection, as the Circle *ZHNO*, Fig. 1. or the Circle *NWSE*, Fig. 2. &c. in Plate 2.

All the rest of the great Circles in any Projection, are either right Circles, or oblique Circles.

A right Circle appears to the Eye to be a right Line, cutting the primitive Circle at opposite Points, and passing through the Center thereof, thereby dividing the Superficies of it in two equal Parts, as in Fig. 1. the Lines *PaS* and *ÆaQ* and *EaC* are right Circles.

An

An oblique Circle cuts the Primitive in opposite Points, but does not pass through the Center of it, as in *Fig. 1.* the Circles $P \odot S$, and $Z \odot N$, are oblique Circles.

A Parallel, or lesser Circle, neither cuts the Primitive at opposite Points, nor (except accidentally) passes through its Center, as the Circle $b \odot d$, *Fig. 1.* is a lesser Circle, viz. a Parallel of Altitude, or an Almicanthar.

S E C T. II.

How to project and measure any of the aforesaid Circles Stereographically, which may properly be called Spherical Geometry.

P R O B L E M I.

How to find the Pole of any great Circle.

Note, **T**HE Pole of a great Circle is a Point every Way 90 Degrees distant from its Periphery, for every great Circle contains 360 Degrees, and if two great Circles cut each other at right Angles, (viz. one Perpendicular to the other) there will be 180 Degrees of one Circle on each Side of the other, and the half of that, viz. 90 Degrees number'd either Way from the

Periphery of any great Circle, finds its Pole; thus the Zenith and Nadir are the Poles of the Horizon, and the Poles of the World are the Poles of the Equator, &c.

C A S E I.

To find the Pole of the Primitive Circle.

Find its Center, and that is its Pole required.

C A S E II.

To find the Pole of a right Circle, suppose of the right Circle $\mathcal{A}EaQ$.

Cross the Diameter $\mathcal{A}EaQ$ at right Angles, with the Diameter $p a S$, the two
Plate 2. Points p and S at a Quadrant's Di-
Fig. 1. stance from $\mathcal{A}E$ and Q are the Poles of the Right Circle, $\mathcal{A}EaQ$ required.

C A S E III.

To find the Pole of an oblique Circle, suppose of the Circle $Z \odot N$.

Note, The Pole of an oblique Circle is always in that Diameter that passes through its Center, and cuts it in the Middle at right Angles: Thus the Pole of the oblique Circle $Z \odot N$ is in the Diameter $H a O$, and is found thus.

Through a the Center of the primitive, and O the Center of the oblique Circle, (which in this Case falls without the primitive) draw the
the

the Diameter HaO ; lay a Ruler from Z to R , and it cuts the primitive in h ; from h set off 90 Degrees to K , and then a Ruler laid from K to Z cuts the Diameter HaO in L ; the Pole of the Circle $Z \odot N$ required, and (Fig. 2.) the Pole of the oblique Circle wqe , is in the Point r , &c.

PROBLEM II.

To find the Center of any given oblique Circle, if it be known what Angle it makes with the primitive. (Plate 2. Fig. 1.)

To find the Center of the oblique Circle ZRN , which makes an Angle with the primitive Circle at Z of $50^{\circ} 0'$.

VARIETY I.

With the Tangent of the given Angle, and one Foot in a , the Center of the primitive, the other Foot extended along the Diameter HaO , (continued if need be) will reach to the Point o , the Center required.

Or thus.

VARIETY II.

The Point K being found by Prob. 1. Case 3: the Extent NK set from K to y , a Ruler laid from z to y , cuts the Diameter HaO in o , the Center required.

VARIETY III.

Or it may be found without having the Diameter HaO , thus $\longrightarrow$ with the Secant of the given Angle at the primitive, and one Foot in N , sweep the small Arch o , with the same Extent; and one Foot in Z , cross the said Arch in o , that Crossing is the Center required.

PROBLEM III.

To project any given Angle. (Plate 2. Fig. 1.)

CASE I.

To project any given Angle (suppose of $23^{\circ} 30'$) at the Center of the primitive.

Having drawn the primitive, and crossed it at right Angles, with the Diameters PaS , and $\mathcal{A}aQ$, set the Chord of the given Angle $23^{\circ} 30'$, from $\mathcal{A}$ to E , and through E and a , draw the Diameter EaC , then shall $\mathcal{A}aE$, or BaO , be the Angle required.

CASE II.

To make any given Angle at the Periphery of the primitive, as suppose I would make at the Point Z an Angle with the primitive of $50^{\circ} 0'$.

Find the Center O , by Prob. 2. Variety 2 or 3, then with one Foot in o , and the other in z or n ; draw the Circle $Z \odot N$, and 'tis done;

done; and the Angle HZK is the Angle required.

CASE III.

To make an oblique Circle; to make at the primitive any given Angle with another oblique Circle.

Having given the Circle WqE , making an Angle with the primitive $35\ 20$, it is required to draw another oblique Circle, to make at W an Angle of $23\ 30$, with the oblique Circle WqE . Plate 2.
Fig. 2.

Lay a Ruler from W to q , it shall cut the primitive in b ; then set off the Chord of the given Angle $23\ 30$, from b to a . A Ruler laid from a to W , cuts the Diameter NS in C , then from E through C to W , draw an oblique Circle, and 'tis done.

Thus you may make any Angle, though you do not know what Angle the given Circle WqE makes with the primitive; but if it be known what Angle the given oblique Circle makes with the primitive, you may find the Center of the required oblique Circle by *prob. 2. Var. 3.* As here the Circle WqE is supposed to make an Angle of $35\ 20$, with the primitive at W , and the Circle required to be drawn is to make at W an Angle of $23\ 30$, which added to $35\ 20$, the Sum is $58\ 50$, the Angle which the Circle WcE is to make with the Primitive; therefore with the Secant of $58\ 50$, and one Foot in W , and E , find the Center by *prob. 2. Var. 3.* which falls without.

CASE IV.

To project any given Angle within the Periphery, but not at the Center.

VARIETY I.

To draw an oblique Circle through any given Point in a Right Circle, to make with the said Right Circle any given Angle.

It is required to draw an oblique Circle through the Point $\odot$ in the Right Circle $Q \odot R$, to make an Angle at $\odot$ of $50^\circ 00'$, with the said Right Circle.

Find the Poles of the Right Circle by *Prob. 1. Case 2.* which here are the Points g and h .

Lay a Scale from g to $\odot$, it cuts the primitive in K , the Arch QK measured on the Chords, is $20^\circ 12'$, therefore take the Tangent of $20^\circ 12'$, and set from o (the Center of the primitive) to c , and draw $c \perp$ perpendicular to $Q R$, so is $c \perp$ continued a Line, in which the Centers of all oblique Circles fall that pass through the Point $\odot$ ---- Then with the Chord of 60 , and one Foot in $\odot$, describe the Arch $a b$, upon which from a set off the Chord of the Complement of the Angle required, which is $40^\circ 0'$ (because the required Angle is $50^\circ 0'$) to b . a Scale laid from $\odot$ to b , cuts the perpendicular $m n$ in x , then upon x as a Center with the Distance $x \odot$, draw the oblique Circle $S \odot T$, and 'tis done; and the Angle $S \odot Q$ is the Angle $50^\circ 00'$ required.

VARIETY

VARIETY II.

Through any given Point in an oblique Circle, to draw another oblique Circle, to make with the Former any given Angle, *Plate 1. Fig. 4.*

Through the Point $\odot$, in the oblique Circle $S \odot T$, it is required to draw another oblique Circle, to make an Angle of $27\ 19$, with the said given Circle.

Through the Point $\odot$ and the Center q , draw the Right Circle $Q q R$, then measure the Angle $Q \odot S$, by *Prob. 10. of Spherical Geometry*, which will be found to be $50\ 00$; to which add the Angle required to be made $27\ 19$, the Sum $77\ 19$ is the Angle that the oblique Circle is to make with the aforesaid Right Circle $Q a R$, and is projected as in the last Variety, its Center being thereby found to be in the Point r , and $y \odot z$ is the Circle required.

Note, If the Nature of the Question had required the Circle to be projected to lye nearer the Right Circle, than the given oblique Circle, you must have subtracted the Angle required to be made from the Angle, that the given oblique Circle makes with the Right Circle; the Remainder is the Angle that the required oblique Circle is to make with the Right Circle.

EXAMPLE.

It is required through the Point $\odot$, in the oblique Circle $y \odot z$, to draw another oblique Circle, to make with the former an Angle of $27\ 19$. *Plate 1. Fig. 4.*

Having

Having drawn the Right Circle $Q \odot R$, through the Point $\odot$, and measuring the Angle $y \odot Q$, it will be found to be $77\ 19$; from which substract the Angle required to be made, viz. $27\ 19$, the Remainder $50\ 00$ is the Angle, that the Circle to be projected is to make with the Right Circle, and may be perform'd by the foregoing Rules.

PROBLEM IV.

To draw a great Circle through any given Point to make with the primitive any given Angle.

Through the Point n , to describe *Plate 2.* a Circle, to make with the primitive *Fig. 2.* an Angle of $35\ 20$.

With the Tangent of $35\ 20$, and one Foot in the Center y , describe the Arch x ; then with the Secant of $35\ 20$; and one Foot in the given Point n , cross the said Arch at x , that Crossing is the Center of the Circle required; then with the Distance $x\ n$, draw the Circle $w\ n\ e$, it makes at w , the Angle $35\ 20$, with the primitive, as was required.

Note, When you sweep with the Tangent of the given Angle, and one Foot in the Center, you may carry the Sweep round, and then it will be easier to find where the Secant cuts it.

P R O B L E M V.

Through any two Points, (as $\odot$ and Z) to draw a great Circle.

Through either Point, as suppose the Point $\odot$, and the Center of the primitive, draw the Diameter $Q\odot a R$ continued, *Plate 2.* and cross it at Right Angles, with the *Fig. 5.* Diameter $h a g$.

From the Point $\odot$ draw a Line to the Extremity of the second Diameter, as the Line $\odot g$.

At the End of that Line, viz. at g erect the Perpendicular $g y$, to cut the Diameter $Q R$ in y , then through the Points $y z$ and $\odot$, draw a great Circle, and 'tis done.

Or because there will always be 180 of the half Tangents, between y and $\odot$ having drawn the Diameters $g h$, and $Q d$ continued, produce $g \odot$ to b , lay a Ruler from b to a , it will cut the primitive in c ; then a Ruler laid from g to c , cuts $Q R$ in y , the third Point through which the proposed Circle is to pass.

Or measure $a \odot$ from the Beginning of the half Tangents, which being found, subtract it from 180; the Remainder taken from the half Tangents will reach from a to y , the third Point required, and the Points $y z \odot$ is brought into an Arch of a Circle, by the seventh Geometrical Problem.

P R O B L E M VI.

To draw a great Circle perpendicular to a given great Circle.

Note, Any great Circle, whether Right or Oblique, drawn through the Poles of another great Circle, is always perpendicular to that great Circle, through whose Poles it is drawn.

C A S E I.

To draw a great Circle perpendicular to the primitive.

Draw a Diameter through the Center of the primitive, and 'tis done ; thus ZN and HO are perpendicular to the primitive.

Plate 2.

Fig. 1.

C A S E II.

To draw a Right Circle perpendicular to a Right Circle given.

It is only to cross the given Right Circle at Right Angles with another Right Circle, and then the Extremities of the one are the Poles of the other ; thus the Right Circle $Ha o$ is perpendicular to the Right Circle $Za N$.

C A S E III.

To draw an oblique Circle perpendicular to a Right Circle.

Find

Find the Poles of the given Right Circle, by *prob. 1. Case 2.* then through these two Poles draw an oblique Circle, and 'tis done; — thus the oblique Circle $Z R n$ is perpendicular to the Right Circle $H a Q$, because cutting its Poles z and n .

CASE IV.

To draw an oblique Circle perpendicular to a given oblique Circle; suppose to the oblique Circle $w q E$: Find the Pole r by *prob. 1.* and through r draw the oblique Circle $m o z$, it is the Circle required. *Plate 2. Fig. 2.*

Note, If the Point o be given, draw an oblique Circle through the Points o and r , by *Prob. 5.* Or if the Angle with the primitive be given, draw the Circle through the given Point r , by *Prob. 4.*

PROBLEM VII.

To project a parallel Circle.

In this Problem are three Varieties.

I. To lay down a Parallel to the primitive Circle, suppose $20^{\circ} 12'$ distant from it, or $69^{\circ} 48'$ from its Pole, with the half Tangent of $69 48$ its Distance from the Pole of the primitive, and one Foot in the said Pole at o , sweep the Circle $\odot uv$, it is the Parallel required. *plate 2. Fig. 4.*

II. To

II. To lay down a Parallel to a Right Circle at 20 12 Distance from it; set 20 12 of the Chords from g to m , and from h to o ; then with the Tangent Complement of 20 12, viz. 69 48, and one Foot in m , (or with the Secant of 69 48, and one Foot in a) find the Point P , then with the Extent $P m$, and one Foot in P , draw the Parallel $m n o$, it is the Parallel required.

III. To draw a Circle parallel to the oblique Circle $a b c$, at 40 12 Distance from it, or 49 48 from its Pole.

Find the Pole of the oblique Circle, by *Prob. 1.* (which will also be the Pole of the Parallel) which here is at d , set off the half Tangent of 49 48, from d to r and s , the Distance $r s$, bisected in e , gives the Center of the Parallel, and with the Distance $e r$, or $e s$, describe the Circle $r s t$, it is the Parallel required.

Or because the half Tangents are projected, by laying a Ruler from a to the equal Divisions of the Semi-circle $a n c$, and where these cut the Diameter, $i n$ continued are the half Tangents; therefore setting 49 48 of the Chords from K , (the Point where a Ruler laid from a over the Pole d , cuts the primitive) both Ways to m and z , a Ruler laid from a to m , and z cuts the Diameter $i n$ in r and s , which bisected in e , finds the Center required.

PROBLEM VIII.

To measure an Arch of a Circle.

General Rule.

In all great Circles, whether right or oblique, (for the primitive Measures on the Chords without farther Application) a Ruler laid from the Poles of the Circle, to be measured, over the Extremities of the Arch to be measured, and Marks made where the Ruler so laid, cuts the primitive, the Distance between those Marks measured on the Chords, gives the Quantity of the Arch required.

EXAMPLE.

1. It is required to measure the Arch Pz of the primitive Circle $z o n H$.

Take the Extent $z P$ in your Compasses, and apply it to the Line of Plate 2. Chords, it gives 35 20, the Quantity Fig. 1. of the Arch required.

2. To measure the Arch $a L$ of the Right Circle $H a o$.

From z the Pole of the Right Circle $H a o$, lay a Scale over L , it will cut the primitive in K ; the Arch of the primitive $z K$ measured upon the Chords, is 50^d the Arch $a L$ required.

3. To

3. To measure the Arch of an oblique Circle. It is required to measure the Arch qn of the oblique Circle wqE .

Find the Pole r by *Prob. 1.* then from r lay a Ruler over n , it will cut the primitive in a , the Arch sa measured upon the Chords, is $64^{\circ} 0'$, the Quantity of the Arch qn required.

plate 2.

Fig. 2.

PROBLEM IX.

To set off any Number of Degrees upon a given Circle from a given Point.

This is only the Reverse of *Prob. 8.* for laying a Ruler from the Pole of the given Circle over the given Point make a Mark, where it cuts the primitive, set the Chord of the given Number of Degrees from that Mark upon the primitive, and from the Extremity of that Extent to the Pole of the given Circle, lay a Ruler, and where that cuts the given Circle, determines the Degree required.

EXAMPLE.

It is required to set off 64 Degrees from q toward E , upon the oblique Circle wqE .

From the Pole r , lay a Ruler over q , it cuts the primitive in s , from s set off 64 of the Chords to a , a Ruler laid from the Pole r to a , cuts the oblique Circle wqE in n , the Arch qn is 64 Degrees, the Quantity required to be set off from q .

By

By the same Method of laying a Ruler from the Pole to the primitive, you may set off any Number of Degrees upon a Right Circle, as in Fig. 1. If I would set off 50 Degrees from a , upon the Right Circle $Ha o$; if you lay a Ruler from z over a , it cuts the primitive in n , set the Chord of 50 from n to K , a Ruler laid from z to K , cuts $Ha o$ in L ; then is $a L$ 50 Degrees of the Right Circle required, or it may be set off by the Scale of half Tangents.

The primitive is always measured upon the Chords.

PROBLEM X.

To measure a Spherical Angle.

General Rules.

An Arch of a Circle passing between the two Sides, containing the Angle, and at a Quadrant's Distance from it, (of which consequently the Angular Point must be the Pole) measured by Prob. 8. is the Measure of the Angle required. — Or

The Distance between the Poles of the Circles, including the Angle, is the Quantity of the Angle required; and these General Rules hold true in all Cases, whether Primitive, Circle, Right, or Oblique.

In this Problem are four Varieties. Plate 2.

Fig. 1.

I. A Right Circle with the primitive makes always Right Angles, as zn , and Ho , with the primitive $z H n o$.

II. A

II. A Right Circle with a Right Circle, and here because the Poles of all Right Circles are in the primitive, the Distance between their Poles, is only measured thereon.

I desire to know the Quantity of the Angle $\text{Æ} a H$. The Pole of $\text{Æ} Q$ is P , and the Pole of HO is z , and $z P$ measured on the Chords is $35\ 20$, the Angle $\text{Æ} a H$ required.

III. A Right Circle with an oblique Circle, as suppose it is required to know what Angle the Right Circle $N S$ makes with the oblique Circle $Z r m$.

Find the Pole of the Right Circle E , and of the oblique Circle r , by *Prob. 1*. A Ruler laid from the Angle r , to the Pole of the oblique Circle r , will cut the primitive in l , and E being the Pole of the Right Circle; the Arch $l E$ measured on the Chords $50\ 00$, is the Quantity of the Angle r required.

IV. An oblique Circle with the primitive.

The Angle $q w s$ is required to be measured.

The Pole of the primitive is the Center y ; the Pole of the oblique Circle $w q E$ is at r .

A Ruler laid from the Angular Point w to y , cuts the primitive in E , and from w to r , cuts it in d ; the Arch $d E$ measured on the Chords is $35\ 20$, the Quantity of the Angle $q w s$ required.

V. An

V. An oblique Circle with an oblique Circle ;
the Angle $Z \odot P$ is required.

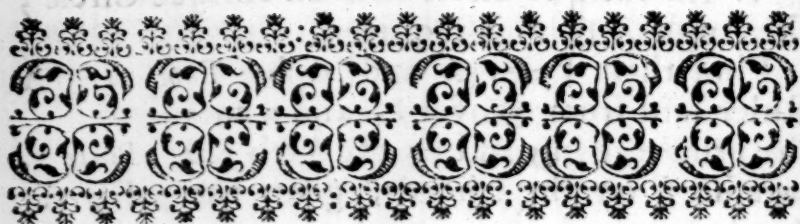
Find their Poles L and x , by *Prob. 1.* A
Ruler laid from $\odot$ over L , and x cuts
the primitive in y and w ; the Arch yw *Plate 2.*
measured on the Chords, gives the *Fig. 1.*
Angle required.

Note, All these *Varieties* come under the first
General Rules ; only we have given an Instance
in each, to make them more intelligible.



D

CHAP.



C H A P. III.

Of Spherical Trigonometry.



S E C T. I.

D E F I N I T I O N S.

Plate 2. Fig. I.



THE several Circles and Points given, or required, being projected as before is taught; any three great Circles intersecting each other, the Arches comprehended between the three Points of Intersection, do constitute a Spherical Triangle; and such Triangles, constituted of Arches, of any three great Circles
of

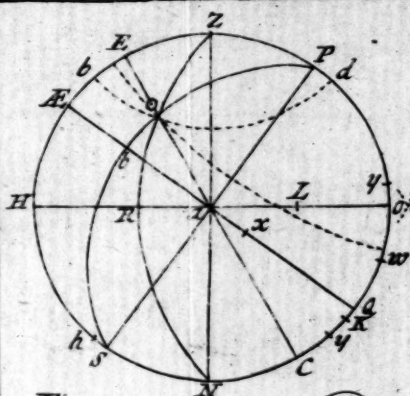


Fig. 1.

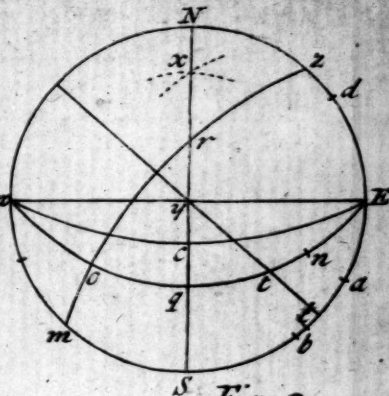


Fig. 2.

Plate 2.

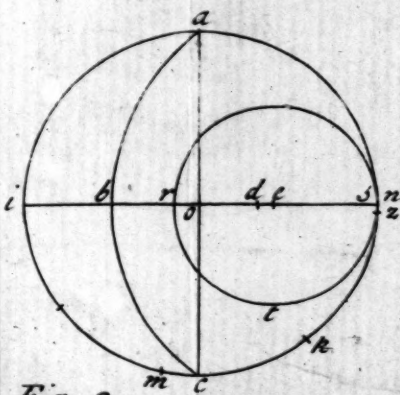


Fig. 3.

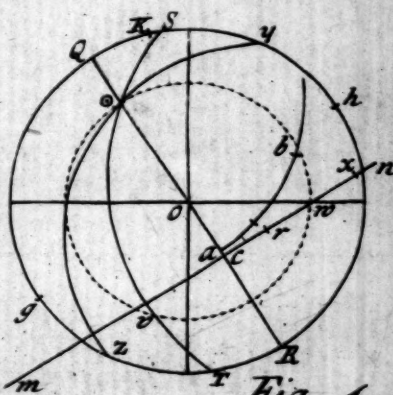


Fig. 4

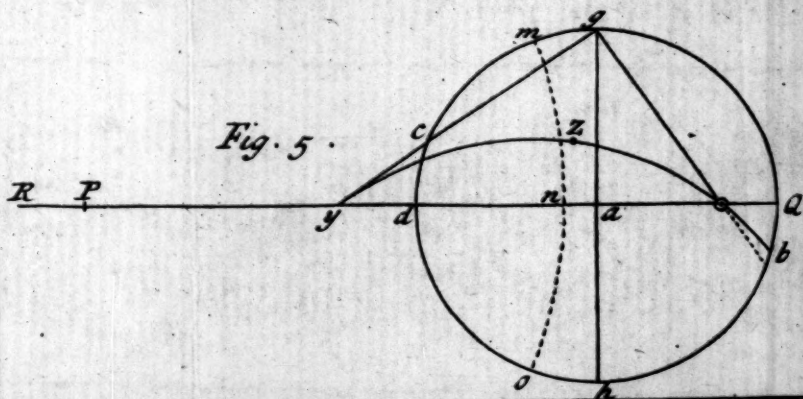


Fig. 5.

of the Sphere, are the Subject of Practical Spherical Trigonometry: As in *Fig. 1.* the Circles $E \odot a c$, and $\mathcal{A} b a Q$, and $P \odot b S$, do cut each other in the three Points $a b \odot$; the three Arches $a b$, $b \odot$, and $\odot a$, making up the Triangle $a b \odot$.

Of Spherical Triangles, are three Sorts, viz. Right-angled, Oblique-angled, and Quadrantal.

A Right-angled Triangle, hath one Angle 90 Degrees: As the Triangle $a b \odot$, Right-angled at b , *Fig. 1.*

A Quadrantal Triangle, hath one Side 90 Degrees: As the Triangle $a \odot z$, *Fig. 1.* whose Quadrantal Side is $a z$.

An Oblique Triangle, hath no Side, nor Angle 90 Degrees: As the Triangle $Z P \odot$, *Fig. 1.*

Of an Oblique Triangle, some of the Angles are acute, and some obtuse.

An Acute-angle, is less than 90 Degrees: As the Angle $Z P \odot$, *Fig. 1.*

An Obtuse-angle, is more than 90 Degrees: As the Angle $P Z \odot$, *Fig. 1.*

NOTE, When two Letters are express'd, they signify the Side contain'd between them; but when three are express'd, they signify the Angle at the middle-most Letter: Thus the Angle $P Z \odot$, signifies the Angle at Z , in the Triangle $P Z \odot$, &c.

S E C T. II.

S O L U T I O N S.

In Spherical Triangles observe these Solutions.

1. **T**HAT the Perpendicular and Base of a Right-angled Spherical Triangle, are of the same Affections with their opposite Angles, viz. More or Less than a Quadrant, as their opposite Angles are.

2. If the Legs (and consequently the Angles) of a Right-angled Triangle, be both More, or both Less than a Quadrant, the Hypothenuse is less than a Quadrant; but if of different Affections, viz. one More, and the other Less, the Hypothenuse is more than a Quadrant.

3. Consequently, if the Hypothenuse be less than a Quadrant, the Legs are of the same Affection; if more, they are different.

4. If the Hypothenuse be less than a Quadrant, each Leg shall be of the same Affection with its adjacent Angle; but if more, they are of different Affections.

5. Conse-

5. Consequently if a Leg be of the same Affection with its adjacent Angle, the Hypotenuse is less than a Quadrant, and the contrary.

6. If you describe a great Circle at 90 Degrees Distance from any Angle, the Arch of this Circle intercepted between the Legs of the Angle, is the Measure of the Angle.

7. The Poles of the three Sides of any Triangle, constitute another Triangle; for the three Angles of this Supplemental Triangle, are made by the mutual Interfection of the Circles, of which the Sides are formed in the Triangle, whose Sides are given that are equal thereunto.

NOTE, In this Case you use the Complement of the greatest Side, or the greatest Angle, to a Semicircle.

8. In an Oblique Spherical Triangle, if the Angles at the Base be both of the same Affection, the Perpendicular let fall from the Vertical Angle, falls within the Triangle; but if different, it falls without.

9. When an Oblique Spherical Triangle is to be divided into two Right-angled Ones, by a Perpendicular, you must let the Perpendicular fall from the End of a given Side, and opposite to a given Angle.

10. There are twenty eight Cases in Spherical Triangles, viz. Sixteen in Right-angled, and Twelve in Oblique; all which may be solved by the Axioms and Rules following.

S E C T. III.

AXIOMS and CONCLUSIONS.

A X I O M L.

THE Sine of Middle-part, and Radius, are reciprocally proportionable to the Tangents of the Extrems-conjunct; and to the Sine-complements of the Extrems-disjunct:

That is, As Radius to Tangent of one Extream-conjunct; so the Tangent of the other Extream-conjunct, to the Sine of the Middle-part.

And as Radius to Sine-complement of one Extream-disjunct; so the Sine-complement of the other Ext. Disj. to Sine of Middle-part.

But if Middle-part be given, and one of the Extrems required, the Proportion may be inverted to bring always the required Term into the last Place thus:

As Sine-complement of the given Extream-disjunct, is to Radius; so is the Sine of Middle-part, to the Sine-complement of the Ex-

tream-

treame-disjunct required: And the like in Extreame-conjunct, as a little Practice will make plain.

AXIOM II.

In all Spherical Triangles, the Sines of the Sides are proportionable to the Sines of their opposite Angles, and the contrary.

These two Axioms, with the Help of the following Conclusions, or Rules, are sufficient for solving all the twenty-eight Cases of Spherical Triangles.

1. As the Sine of the Base to Radius; so the Tangent of the Perpendicular, to the Tangent of the Angle, at the Base of a Right-angled Triangle.

2. In all Right-angled Triangles, as the Sine of the Hypothenuse to Radius; so is the Sine of the Perpendicular, to the Sine of the Angle at the Base.

These two Conclusions are thus demonstrated by Mr. Norwood; For the First, Let $a d B$ be a Spherical Triangle, *Plate 3.* Right-angled at B ; then is $a d$ the *Fig. 3.* Sine of the Hypothenuse, $a B$ the Sine of the Base, $a M$ is Radius, and $M L$ the Tangent of the Angle at a , and $B k$ the Tangent of the Perpendicular $B d$: Then;

As $a B$ to $a M$; so $B k$ to $M L$ by *Euc.*
Lib. 6. Prop. 4.

For the Second, $a d$ is the Sine of the Hypothenufe, and $a I$ is Radius, $d c$ is the Sine of the Perpendicular $d B$, and $H I$ is the Sine of the Angle at a : Then;

As $a d$ to $a I$; so $d c$ to $I H$ by *Euc.* *Lib. 6. Prop. 4.*

3. The Sines of the Angles subtended by the Perpendiculars, are directly proportional to the Sines of the Perpendiculars subtending.

4. If two Perpendiculars subtend equal Angles, the Sines of the Perpendiculars are directly proportional to the Sines of the Hypothenufes.

 5. The Co-sines of the Angles at the Base, are directly proportional to the Sines of the Vertical Angles.

 6. The Co-sines of the Segments of the Base, are directly proportionable to the Co-sines of the Hypothenufes.

 7. The Sines of the Bases, are reciprocally proportional to the Tangents of the Angles at the Base.

 8.

 8. The Tangents of the Sides, are reciprocally proportional to the Co-sines of the Vertical Angles.

9. The Tangents of the Bases, are proportional to the Tangents of the Vertical Angles.

10. Having let fall the Perpendicular, as before, it will be as the Sine of the Sum of the Hypothenuses, to the Sine of their Difference; so the Co-tangent of half the Sum of the Vertical Angles, to the Tangent of half their Difference; and so the Co-tangent of half their Difference, to the Tangent of half their Sum.

11. As Tangent of half the Sum of the Sides or Hypothenuses, to Tangent of half their Difference; so Tangent of half the Sum of the Angles at the Base, to Tangent of half their Difference.

 12. As Tangent of half the Base, to Tangent of half the Sum of the Sides; so Tangent of half the Difference of the Sides, to Tangent of half the Difference of the Segments of the Base; which half Difference, added to the half Sum, gives the greater Base, and subtracted, gives the lesser.

These two Axioms, and twelve Conclusions, I thought fit to insert for Variety: Nevertheless, all the twenty eight Cases of Spherical

rical Triangles may be solved by the two Axioms, and the five Conclusions marked thus , viz. The 5, 6, 7, 8 and 12; for the sixteen Cases in Right-angled Triangles, are solved by the first Axiom: The first and fourth Cases in Oblique, are solved by the second Axiom: The Second, Third, Fifth, Sixth, Seventh, Eighth, Ninth and Tenth in Oblique, are solved by Axiom the first, with the Help of the Conclusions above-mention'd, viz. The 5, 6, 7 and 8; provided you observe in your first Operation, to find such a Part, which when found, the other Part of the Thing required, which is to be added to, or subtracted from the first Part found, may come under one of the four Conclusions above-mention'd; and the eleventh Case where three Sides are given, and the twelfth Case where the three Angles are turned into Sides, are both solved by Conclusion the twelfth, and Axiom the first, only in letting fall a Perpendicular, in this Case, it must always fall from the greatest Angle, upon the greatest Side; and so the lesser Sides become Hypothenuses to two Right-angled Triangles, which being given, and the Bases found by Conclusion 12, you have enough to find with Ease any other Part of either Triangle; but for the Sake of those that have learned another Way, and are not willing to leave it, I shall insert the following Axiom, which we may call,

These two Axioms and twelve Conclusions I thought fit to insert for Variety: The twenty-third Case of Spherical
A X I O M

AXIOM III.

As the Rect-angle of the Sines of the Sides containing the Angle required, to the Square of Radius; so is the Rect-angle of the Sines of half the Sum of the three Sides, and of the Difference of the Side opposite there-from, to the Square of the Sine-complement of half the Angle sought.

From hence we may deduce this Rule for Operation.

Add the three Sides together, and from their half Sum, subtract the Side opposite to the Angle required.

Then to the Complement Arithmetical of the Logarithm-sines, of the containing Sides, add the Logarithm-sines of that half Sum, and Remainder; half the Total of these four Logarithms is the Sine-complement of half the Angle required.

For the better understanding of my Lord Neſair's Universal Proportion, laid down in the first Axiom, you must understand, That in every Triangle there are six Parts, viz. Three Sides and three Angles; but omitting the Right-angle, there are but Five, viz. The three Sides, and the two Plate 3. Acute-angles, of which the two Legs, Fig. 2. including the Right-angle, and the Complements of the Hypothenuſe, and two

Acute-angles; are called the five Circular-parts, of which there are always Two given, to find a Third; and of these Three, viz. Two given, and One required; One is always Middle-part, and the other Two are either Extreams-conjunct, or Extreams-disjunct; for if the three Parts lie together, the Middle-most is Middle-part, and the other Two are Extreams-conjunct, as in the Figure annex'd: Suppose the Angles a and c , were given to find the Hypothenuse $a c$, than $a c$ is Middle-part, and the other two Parts, viz. the Angles a and c , are Extreams-conjunct; and by Axiom the first, the Proportion should be as Radius to Tangent, of one Extream-conjunct; so the Tangent of the other Extream-conjunct, to the Sine of Middle-part: But because in this Case, the Question falls amongst these Parts that are noted by their Complements, you must instead of Tangent, use Tangent-complement, and instead of Sine, use Sine-complement (or in Case they had been Extreams-disjunct, you must instead of Sine-complement, have used Sine) and then the Proportion is as Radius to the Tangent-complement of the Angle at a , one Extream-conjunct; so the Tangent-complement of the Angle at c , the other Extream-conjunct, to Sine-complement of $a c$ the Middle-part.

But if Middle-part be given, you must alter the Proportion; for the required Term must always be the last in the Proportion: As suppose the Angle at a , and the Hypothenuse $a c$, were given to find the Angle at c ; here $a c$ is Middle-

Middle-part, as before; but because it is an Extream-conjunct that is required, the Proportion is as Tangent-complement of a (the given Extream-conjunct) to Radius; so the Sine-complement of $a c$ (the Middle-part) to Tangent-comp. of c , the required Extream Conj. And thus however the three Parts that enter the Question be dispos'd of, you may always find such an One to make Middle part, as that the other Two will either be both Extreams-conjunct, or Extreams-disjunct, viz. If the three Parts lie together, the Middle is Middle-part, and the other two Extream-conjuncts; but if one lie remote, it is Middle-part, and the other Extream-disjuncts, as in the following Table.

If	$a b$	be Middle-Part,	Comp. a	are Ext. Conj. and	Comp. c	are Ext. Disj.
	$b c$		$b c$		Comp. $a c$	
			$a b$		Comp. a	
	Comp. c		Comp. c		Comp. $a c$	
			$b c$		Comp. a	
			Comp. $a c$		$a b$	
	Comp. $a c$		Comp. a		$a b$	
			Comp. c		$b c$	
	Comp. a		Comp. $a c$		Comp. c	
			$a b$		$b c$	

Quadrantal Triangles, are also wrought by the same Laws of Right angled Triangles, allowing the Quadrantal-side in one, to be the same as the Right-angle in the other; and as in Right-angled Triangles, the Legs, or Sides, next the Right-angle, and the Complements of the

the Hypothenuſe, and Acute-angles, are the five Circular-parts; ſo in Quadrantal Triangle, the Quadrantal-ſide is omitted, and the two Angles, adjacent to the Quadrantal-Side, and the Complements of the other two Sides, and of the Angle oppoſite to it, make the five Circular-parts, and come under the Rules of the Catholick Proportion: Of which ſee an Example in *Prob. 24.* of Astronomy, performed by Right-angled Triangles; and in *Prob. 15.* of Oblique: But for the better Underſtanding hereof, Suppoſe the Triangle abc , to be a Quadrantal Triangle, viz. The Side ab 90 Degrees, then is the Angle a , and the Angle b , with Complement of the Side ac , and Complement of the Angle c , and Complement of the Side cb , the five Circular-parts; and ſo in all others.

Plate 3.

Fig. 1.



SECT.

S E C T. IV.

The Globical Projection of the Sphere:

THERE is also another Way to find the Proportions for solving the sixteen Cases of Right-angled Spherical Triangles, by the or Globical Projection of the Sphere: Of which take this brief Account (*Plate 3; Fig. 4.*)

With the Chord of 90 Degrees, and one Foot of the Compasses any where, as in *A*, draw the Circle *O H K d*; then with the same Extent, and one Foot any where in the Periphery of that Circle, as in *O*, draw the Circle *A R F G*, set the Chord of the Angle at *A* (as suppose 23. 30.) from *O* to *Q* (although in this you need not be exact; for you may find the Proportion as well, if you make the Angle *A* at Pleasure) then with the Chord of 90, and one Foot in *Q*, sweep the Circle *A E F N*; and because the Triangle is to be Right-angled at *B*, take the Chord of 90, and setting one Foot at some convenient Distance from *A* (as suppose at *B*) with the other Foot find the Point *m*, in the same Circle *A E F N*, upon which the Right-angle is to be, and upon *m*, as a Center, describe the Circle *B C P G H*, then is the Triangle *A B C* finished; then with the same Extent, and one Foot in *C*, draw

draw the Circle $O I K E$, and then is the Projection fitted for our intended Use.

This being done, observe these four Rules; First, What Parts of your Triangle is given: As suppose the Hypothenufe AC , and the Angle at A , to find the Leg AB ; I mark the Side AC , and also the Angle at A , with a Dash thus $'$, and the Required-leg with this Mark o ; then because AC is given, CR is also given, being its Complement to 90 ; because AR is 90 of the Chords, and RS equal to AC , is also given; all which mark for given, as you see, and you may proceed round the Circle if you will, to mark SP , FG , GU , UL , and LA .

The Angle A being given, its Measure Rd , being an Arch 90 Degrees distant from it, is also given; and RQ its Complement, QO equal to RD , and ON its Complement, &c. which mark as before; then because AB is required, BD its Complement is also required, as also Am its Complement, and MN equal to AB , and NT its Complement; all which Mark required, as in the Figure.

2. Then observe in which Mansion your Proportion falls, which is sure to be in that Part of the Projection of which the Triangle is the Corner, drawn here in double Lines, and bounded by the Letters $ABdRSP OQC$. — A Mansion is a Right-angled-triangle, consisting of a Base, a Hypothenufe, and two Perpendiculars; as the Division $ACRd B$ or $BCQRd$, &c. And when Persons

sons are well skill'd in this kind of Projection, they may, instead of the whole Projection, only draw that Part which I have bounded with double Lines; nor need you be careful to have all the Circles, or Parts therein exactly proportionable; but may draw it with a Pen, at Pleasure, holding to the same Form; only always observe to place the Triangle at the bottom Corner, as here you see; and to find your Proportion, observe in which Mansion you have three Things given, and One required, which is here in the Mansion $Q R d B C$, Right-angled at R and d , the Side $Q d$ being the Base, and $Q B$ the Hypothenufe, and $C R$ and $d B$ are Perpendiculars; and here you have $Q R$ given, and also $Q d$ Radius, which is always given; and you have $C R$ also given, to find $B D$ required, in which the Proportion is as in Plain-triangles, viz. As the Short-base $Q R$, to the Long-base $Q d$, or Radius; so the Short-perpendicular $R C$, to the Long-perpendicular $B d$.

3. But to know whether the Proportions lie in Sines, or Tangents, or both, you may observe as in Plain-trigonometry, when the Base is made Radius, the Perpendicular is a Tangent; but when the Hypothenufe is made Radius, the Perpendicular is a Sine; so here, when the Proportion lies in the Base and Perpendiculars, the Long-base is Radius, and the Short-base a Sine, and the two Perpendiculars are Tangents; but when the Proportion lies in the Hypothenufe and Perpendiculars,

lars, the Long-hypothenuſe is Radius, and the Short-Hypothenuſe is a Sine, and the Perpendiculars are Sines.

Fourthly and Laſtly, To know whether your Proportion lies in Sines and Tangents, or in their Complements, obſerve whether the Parts of the Manſion, where the Proportions lies, be the Parts of the Triangle given, or required, or the Complements of thoſe Parts: As in this Caſe in the Manſion $Q d B$, the Proportion is as $Q R$ to $Q d$; ſo $R C$ to $d E$, by Rule the ſecond; but by Rule the third, $Q R$ is a Sine, $Q d$ is Radius, and $R c$ and $d B$ are Tangents; and by Rule the fourth, $Q R$ is the Complement of the Angle at A (becauſe it is the Complement of $d R$ the Meaſure of it) and $C R$ is the Complement of $A C$, and $B d$ is the Complement of $A B$, therefore the Proportion is

As ($Q R$) the Sine-com. of the Angle at A ,	$\left. \begin{array}{l} P. 3. \\ F. 4. \end{array} \right\}$
To ($Q d$) Radius;	
So ($R C$) the Tangent-compl. of $A C$,	
To ($B d$) the Tangent-compl. of $A B$.	

I ſhall for the Learner's further Satisfaction answer all the ſixteen Caſes of Right-angled Spherical Triangles, by this Method; and when I come to the Application of them to Questions in Aſtronomy, I ſhall answer the Triangles by the Proportions found by Middle-parts, and Extreame-conjunct and diſjunct, that nothing may be omitted that is neceſſary for

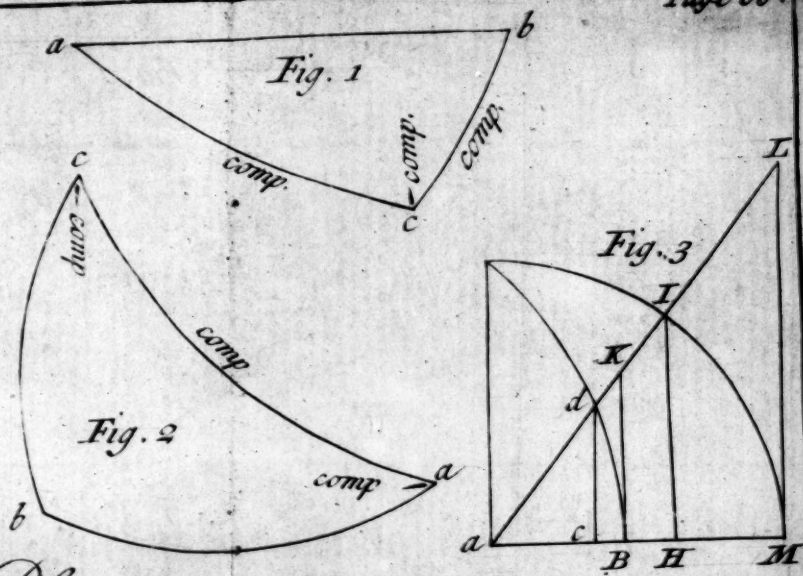
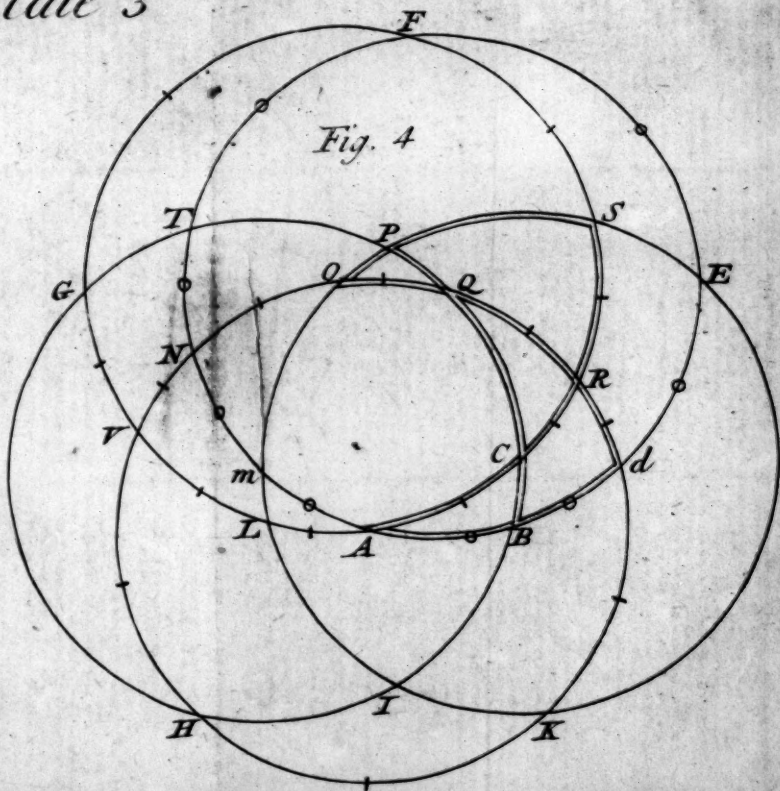


Plate 3



for the Reader's Instruction; nor yet, that I may not needlessly repeat that in Astronomy which has already been sufficiently taught in the Trigonometrical-part.

S E C T. V.

Of Spherical Trigonometry Right-angled.

I Shall here only insert the Operations by the Globical Projection of the Sphere, and leave the Stereographick-projection of them, till we come to the Application of Astronomy, it being the more Practical-part of Spherical-trigonometry, and aiming at Brevity, I shall endeavour to avoid Prolixity on the one Hand, as well as Deficiency on the other.

CASE the first.

Given the Hypothenufe, and an Angle to find the Leg adjacent to the Given-angle.

Given { The Hip. — $60^{\circ}-0'$ } Required a b .
 { Angle a — $23-30$ }

Here

Plate 4. Here the Proportion lies in the
Fig. 1. Mansion $b g d$, Right-angled at d ; and
 hence by the foregoing Rules the
 Proportion is,

As $g e$ Sine-comp. of a , — — 23:30 — 9.96240
 To $g d$ Radius; — — — 90:00 — 10.00000
 So $c e$ Tang. Comp. of $a c$, — 60:00 — 9.76144
 To $b d$ Tang. Comp. $a b$. — 57:48 — 9.79904

CASE the second.

Given the Hypothenufe, and an Angle to
 find the Leg opposite to the given Angle.

Given $\left\{ \begin{array}{l} a c = 60^0 00' \\ a = 23^0 30' \end{array} \right\}$ Required $b c$.

Plate 4. Here the Proportion lies in the
Fig. 2. Mansion $a d e$, and the like in the
 Mansion $b k e$; and either Way it is.

As $a e$, or $b e$ Radius, — — 90:00 — 10.00000
 To $a c$, or $b g$, the Sine of $a c$; 60:00 : 9.93753
 So $d e$, or $e k$, the Sine of a , — 23:30 : 9.60070
 To $b c$, or $g i$, the Sine of $b c$. 20:12 : 9.53823

CASE

CASE the Third.

Given the Hypothenufe, and an Angle, to find the other Angle.

Given $\left\{ \begin{array}{l} ac \text{ --- } 60 \text{ } 00 \\ a \text{ --- } 23 \text{ } 30 \end{array} \right\}$ Required c . Plate 4.
Fig. 3.

The Proportion lyes in the Mansion $ck i$, and is,

As ce Sine Comp. of ac —	60 00 —	9.69897
To ck Radius —	90.00 —	10.00000
So ge Tang. Comp. of a —	23.30 —	10.36170
To ki Tang. of c —	77.44 —	10.66273

CASE the Fourth.

Given the the Hypothenufe and a Leg, to find the Angle between them.

Given $\left\{ \begin{array}{l} ac \text{ --- } 60 \text{ } 00 \\ al \text{ --- } 57 \text{ } 48 \end{array} \right\}$ Required a . Plate 4.
Fig. 4.

The Proportion lies in the Mansion $gd b$, and it is,

As db Tang. Comp. of al —	57.48 —	9.79915
To ce Tang. Comp. of ae —	60.00 —	9.76144
So gd Radius —	90.00 —	10.00000
To ge Sine Comp. of a —	23.32 —	9.96229

CASE

CASE the Fifth.

Given the Hypothenuſe and a Leg, to find the Angle oppoſite to the given Leg.

Plate 4. Fig. 5. Given $\begin{cases} a c & 60 : 00 \\ b c & 20 : 12 \end{cases}$ Required c .

The Proportion lyes in the Manſion $a d e$, or $b k e$, and it is,

As $a c$ Sine of $a c$	_____	60 : 00	—	9.93753
To $a e$ Radius	_____	90 : 00	—	10.00000
So $c b$ Sine of $c b$	_____	20 : 12	—	9.53819
To $d e$ Sine of a	_____	23 : 30	—	9.60066

CASE the Sixth.

Given the Hypothenuſe and a Leg, to find the other Leg.

Given $\begin{cases} a c & 60 : 00 \\ a b & 57 : 48 \end{cases}$ Required $b c$.

Plate 4. Fig. 6. The Proportion lyes in the Manſion $g d b$, and will be,

Rectangled.

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As $d b$ Sine Comp. of $a b$ — 57.48 — 9.72662
 To $c e$ Sine Comp. of $a c$ — 60.00 — 9.69897
 So $g b$ Radius — — — 90 : 00 — 10.00000
 To $g c$ Sine Comp. of $b c$ 20 : 15 — 9.97239

CASE the Seventh.

Given an Angle and a Leg adjacent, to find the Leg opposite to given Angle.

Given $\left\{ \begin{array}{l} a = 23 \quad 30 \\ a b \quad 57.48 \end{array} \right\}$ Reqd. $b c$. Plate 4.
Fig. 7.

The Proportion lyes in the Mansion $a d e$, and it is,

As $a d$ Radius — — — 90 : 00 — 10.00000
 To $a b$ Sine of $a b$ — — — 57 : 48 — 9.92747
 So $d e$ Tang. of a — — — 23 : 30 — 9.63830
 To $b c$ Tang. of $b c$ — — — 20 : 12 — 9.56577

CASE the Eighth.

Given an Angle and a Leg adjacent, to find the Angle opposite to the given Leg.

Given $\left\{ \begin{array}{l} c = 77 : 44 \\ b c = 20 : 12 \end{array} \right\}$ Required a . Plate 4.
Fig. 8.

The Proportion lyes in the Mansion $c k i$, and it is,

As

As $c i$ Radius	—	—	90 : 00	—	10 : 00000
To $i k$ Sine of c	—	—	77 : 44	—	9.98997
So $c g$ Sine Comp. $c b$	—	—	20 : 12	—	9.97243
To $g e$ Sine Comp. of a	—	—	23 : 30	—	9.96240

CASE the Ninth.

Given an Angle and a Leg adjacent, to find the Hypothenufe.

Plate 4. Given $\left\{ \begin{array}{l} c \text{ — } 77 \text{ } 44 \\ b c \text{ — } 20 : 12 \end{array} \right\}$ Required $a c$
Fig. 9.

The Proportion lies in the Manfon $h k e$, and it is,

As $b i$ Sine Comp. of c	—	—	77 : 44	—	9.32728
To $i g$ Tang. of $b c$	—	—	20 : 12	—	9.56576
So $h k$ Radius	—	—	90 : 00	—	10.00000
To $k e$ the Tang. of $a c$	—	—	60 : 00	—	10.23848

CASE the Tenth.

Given an Angle and a Leg opposite, to find the Leg adjacent to the given Angle.

Plate 4. Given $\left\{ \begin{array}{l} a \text{ — } 23 \text{ } 30 \\ b c \text{ — } 20 : 12 \end{array} \right\}$ Required $a b$.
Fig. 10.

The

Rectangled.

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The Proportion lyes in the Mansion *a d e*,
and it is,

As *d e* Tang. of *a* ——— 23 : 30 — 9.63830
To *c b* Tang. of *c b* ——— 20 : 12 — 9.56576
So *a d* Radius ——— 90 : 00 — 10.00000

To *a b* Sine of *a b* ——— 57.48 — 9.92746

CASE the Eleventh.

Given an Angle and a Leg opposite, to find
the Angle adjacent to the given Leg.

Given $\left\{ \begin{array}{l} a \text{ ——— } 23 \text{ } 30 \\ b \text{ ——— } 20 \text{ } 12 \end{array} \right\}$ Required *c*. Plate 4.
Fig. 11.

The Proportion lies in the Mansion *c k i*,
and it is,

As *c g* Sine Comp. of *b c* — 20 : 12 — 9.97243
To *e g* Sine Comp. of *a* — 23 : 30 — 9.96240
So *c i* Radius ——— 90 : 00 — 10.00000

To *i k* Sine of *c* ——— 77 : 44 — 9.98997

CASE the Twelfth.

Given an Angle and a Leg opposite, to find
the Hypothenufe.

E

Plate

plate 4.

Fig. 12.

Give $n \left\{ \begin{array}{l} a = 23.30 \\ b c = 20.12 \end{array} \right\}$ Req. a and d .

The Proportion lies in the Mansion $a d e$, and it is,

As $d e$ Sine of a ————— 23.30 — 9.60070
 To $b c$ Sine of $b c$ ————— 20.12 — 9.53819
 So $a e$ Radius ————— 90.00 — 10.00000

To $a c$ Sine of $a c$ ————— 60.00 — 9.93749

CASE the Thirteenth.

Given the Legs, to find an Angle.

Plate 4.

Fig. 13.

Given $\left\{ \begin{array}{l} a b = 57.48 \\ b c = 20.12 \end{array} \right\}$ Required a .

The Proportion lies in the Mansion $a d e$, and it is,

As $a b$ Sine of $a b$ ————— 57.48 — 9.92747
 To $a d$ Radius ————— 90.00 — 10.00000
 So $b c$ Tang. of $b c$ ————— 20.12 — 9.56576

To $d e$ Tang. of a ————— 23.30 — 9.63829

CASE the Fourteenth.

Given the Legs to find the Hypothenufe.

Rectangled.

75

Given $\begin{cases} a b & 57.48 \\ b c & 20.12 \end{cases}$ Required $a c$. Plate 4. Fig. 14.

The Proportion lyes in the Mansion $g d b$, and it is,

As $g b$ Radius ————— 90.00 — 10.00000
 To $g c$ Sine Comp. of $b c$ ——— 20.12 — 9.97243
 So $b d$ Sine Comp. of $a b$ ——— 57.48 — 9.72663
 To $c e$ Sine Comp. of $a c$ — 60.00 — 9.69906

CASE the Fifteenth.

Given the Angles, to find a Leg.

Given $\begin{cases} a & 23.30 \\ c & 77.44 \end{cases}$ Required $b c$. Plate 4. Fig. 15.

The Proportion lyes in the Mansion $c k i$, and it is,

As $i k$ Sine of c ————— 77.44 — 9.98997
 To $g e$ Sine Comp. of a ——— 23.30 — 9.96240
 So $c i$ Radius ————— 90.00 — 10.00000
 To $c g$ Sine Comp. of $b c$ ——— 20.12 — 9.97243

CASE the Sixteenth.

Given the Angles, to find the Hypothenuſe.

E 2

Given

Plate 4.
Fig. 16. Given $\left\{ \begin{array}{l} a \text{ --- } 23.30 \\ c \text{ --- } 77.44 \end{array} \right\}$ Required $a c$.

The Proportion lyes in the Manſion $c k i$,
and it is,

As $i k$ Tang. of c ——— 77 44 ——— 10.66269
To $g e$ Tang. Comp. of a 23. 30 ——— 10.36170
So $c k$ Radius ——— 90. 00 ——— 10.00000

To $c e$ Sine Comp. of $a c$ 60. 00 ——— 9.69901

S E C T. VI.

Spherical Trigonometry Oblique.

C A S E the First.

G I V E N two Sides and an Angle opposite
to one of them, to find the Angle opposite
to the other, if it be known whether the re-
quired Angle be acute or obtuse.

Plate 4.
Fig. 17. Given $\left\{ \begin{array}{l} a b \text{ } 42 \text{ } 12 \\ b c \text{ } 35 \text{ } 20 \\ c \text{ --- } 32 \text{ } 12 \end{array} \right\}$ Required a .

By

By *Axiom the Second.*

As Sine of $a b$	_____	42 . 12	co. ar.	Q 17282
To Sine of c	_____	32 . 12		7.72663
So Sine of $b c$	_____	35 . 20		7.76218
To Sine of a	_____	27 . 19		9.66163

The Angle at a 27 19 if acute, but if obtuse, it is the Complement of that to 180 Degrees, viz. 152 : 41.

C A S E the Second.

Given two Sides and an Angle opposite, to find the contain'd Angle.

$$\text{Given } \left\{ \begin{array}{l} a b \quad 42 \quad 12 \\ b c \quad 35 \quad 20 \\ c \quad \text{---} \quad 32 \quad 12 \end{array} \right\} \text{ Required } b. \quad \begin{array}{l} \text{Plate 4.} \\ \text{Fig. 18.} \end{array}$$

Let fall the Perpendicular $b d$ by Solution the 9th; and then because the Angle $a b c$ is required, find the Angle $d b c$ by the catholic Proportion, where you find the Side $b c$, is the middle Part, and the other extream Conjuncts, therefore the Proportion inverted is,

E 3

As

As Tang. Comp. of c ———	32 12 —	10.20084
To Radius ———	90.00 —	10 00000
So Sine Comp. of $b c$ ———	35.20 —	9.91158
To Tang. Comp. of $c b d$ —	62.48 —	9.71074

The Angle $c b d$ being thus found, find the Angle $a b d$, by Conclusion the eighth, thus ;

As Tang. of $a b$ ———	42.12	Co ar 0.04252
To Sine Comp. of $c b d$ —	62.48 —	9.66001
To Tang. of $b c$ ———	35.20 —	9.85059
To Sine Comp. of $a b d$ —	69. 4 —	9.55312

The Angle $a b d$ ——— 69 — 4
 Added to the Angle $c b d$ — 62 — 48

The Sum is the Angle $a b c$ 131 — 52

CASE the Third.

Given two Sides, and an Angle opposite, to find the third Side.

Given $\left\{ \begin{array}{l} a b \ 42 \ 12 \\ b c \ 35 \ 20 \\ c \ 32 \ 12 \end{array} \right\}$ Required $a c$. Plate 4.
Fig. 19.

Let fall the Perpendicular $b d$, by Solution the ninth ; and then because $a c$ is required, find $d c$ thus ;

As Tangent-complement of }
 one Extream-conj. bc : } 35.20.10.14941
 To Radius ————— 90 00.10.00000
 So Sine-comp. of Middle-part 32.12. 9.92747
 To Tang. of the other Extr. }
 Conj. dc } 30.57. 9.77806

Then by Conclusion the sixth.

As Sine-comp. of bc — 35.20 Co. ar. 0.08842
 To Sine-comp. of dc — 30 57 — — 9.93329
 So Sine-comp. of ab — 42.12 — — 9.86970
 To Sine-comp. of ad — 38.51 — — 9 89141

The Segment ad ————— 38.51
 Added to the Segment dc — 30.57

The Sum is the whole Base ac 69.48

The second and third Cases may be solved without the Conclusions: As for Instance, In the second Case where the contain'd Angle is required, having two Angles, and their Opposite Sides given or found, the third Angle may be found thus;

As Sine of half the Difference of the Sides
 To Sine of half the Sum of the Sides,
 So Tang. of half the Difference of the Angles
 To Tang. Comp. of half the Angle required.

And in Case the Third, the third Side is thus found ;

As Sine of half the Difference of the given Angles

To Sine of half the Sum of the given Angles,
So Tangent of half the Difference of the Sides
To Tangent of half the Side required.

CASE the Fourth.

Two Angles, and a Side opposite to one of them given, to find the Side opposite to the other, if it be known whether the Side sought be more or less than a Quadrant.

Plate 4.

Fig. 20. Given $\left\{ \begin{array}{l} a b — 42.12 \\ a b c — 131.52 \\ a c b — 32.12 \end{array} \right\}$ Required $a c$

By Axiom the second.

As Sine of c — — — 32.12 Co ar 0.27338
To Sine of $a b$ — — — 42.12 — — 9.82719
To Sine of b — — — 131.52 — — 9.87198

To Sine of $a c$ — — — 69.50 — — 9.97255

NOTE, When a Side, or an Angle sought, is known to be less than a Quadrant, the Sine, or Tangent found, is the Answer to the Question ;

Question; but if more, its Complement to 180 Degrees is the Answer.

NOTE also, When you are to find a Sine, or Tangent, of a Degree above 90, it is found by subtracting the given Degree and Minute, from 180 Degrees; the Remainder is the Sine of the Degree sought: Thus the Sine of 48.8 is the Sine of 131.52, &c.

CASE the Fifth.

Given two Angles; and a Side opposite to one of them, to find the Side included.

Given $\left\{ \begin{array}{l} a = 27\ 19 \\ c = 32\ 12 \\ a\ b\ 42\ 12 \end{array} \right\}$ Required $a\ c$ Plate 4.
Fig. 2.

Let fall the Perpendicular $b\ d$, by Solution the ninth; then because $a\ c$ is required, find the Segment $a\ d$.

As Tang. Comp. of $a\ b$	— 42.12 —	10.04251
To Radius	— 90.00 —	10.00000
So Sine Comp. of a	— 27 19 —	9.94865
To Tangent of $a\ d$	— 38.51 —	9.90614

Then by Conclusion the seventh.

As Tang. of c	—	32.12	Co ar	0.20084
To Sine of $a d$	—	38.51	—	9.79746
So Tang. of a	—	27.19	—	9.71307
To Sine of $d e$	—	30.57	—	9.71137

Then to the Segment $a c$ — 38 51

Add the Segment $d e$ — 30 57

The Sum is the Side $a c$ — 69.48

CASE the Sixth.

Given two Angles, and a Side opposite, to find the third Angle.

Plate 4.

Fig. 22. Given $\left\{ \begin{array}{l} c — 32.12 \\ a — 27.19 \\ b c — 35.20 \end{array} \right\}$ Required b

Let fall the Perpendicular $d b$, by Conclusion the ninth; and then because the Angle $a b c$ is required, find the Angle $d b c$.

As Tang. Comp. of c	—	32.12	—	10.20084
To Radius	—	90.00	—	10.00000
So Sine Comp. of $b c$	—	35.20	—	9.91158
To Tang. Comp. of $c b d$	—	62.48	—	9.71074

Then

Then by Conclusion the fifth.

As Sine Comp. of c — 32 12 Co ar 0.07254

To Sine of $c b d$ — 62 48 — 9 94910

So Sine Comp of a — 27 19 — 9.94865

To Sine of $a b d$ — 69. 3 — 9.97029

CASE the Seventh.

Given two Sides, and an Angle included, to find either Angle.

Given $\left\{ \begin{array}{l} a b \quad 42 \quad 12 \\ b c \quad 35 \quad 20 \\ b \quad 131 \quad 52 \end{array} \right\}$ Required a Plate 4.
fig. 23.

In this Case the Perpendicular may fall either from a or c , by Solution the ninth; but were there is that Variety, observe to let fall the Perpendicular from the End of a given Side, and opposite to a given Angle, and also opposite to the Angle required: As in this Case, or next the Side required, as in Case the ninth.

Then in the Right-angled Triangle $b d c$, Right-angled at d , there is given the Angle $d b c$ (because it is the Complement of the Angle $a b c$, to 180) and the Hypotenuse $b c$, to find the Base $b d$ thus;

E 6

As

As Tang. Comp. of $b c$ ——— 35.20 — 10.14948
 To Radius ——— ——— 90.00 — 10.00000
 So Sine Comp. of $d b c$ ——— 48.8 — 9.82439
 To Tang. of $d b$ ——— ——— 25.19 — 9.67498

Then by Conclusion the seventh.

As the Sine of the Whole- }
 base $a d$ } 67.31 — 0.03433
 To the Tang. of $d b c$ ——— 48.8 — 10.04759
 So Sine of the Segment $b d$ — 25.19 — 9.63106
 To Tang. of a ——— ——— 27.19 — 9.71298

CASE the Eighth.

Given two Sides, and an Angle included, to find the third Side.

Plate 4. Fig. 24. Given $\left\{ \begin{array}{l} a b = 42.12 \\ b c = 35.20 \\ c = 131.52 \end{array} \right\}$ Required $a c$

Here by Solution the ninth, the Perpendicular may fall either from a , upon $c b$ continued, or from c , upon $a b$ continued to d , as here: And then as in Case the seventh, there is by Consequence given the Angle $c b d$, by which, and the Hypotenuse $c b$, you find the Base $d b$: As in Case the seventh, which there, is found to be 25 19

Then

Then by Conclusion the sixth

As Sine Comp. of db	— 25.19	Co ar	0.04386
To Sine Comp. of bc	— 35.20	—	9.91158
So Sine Comp. of ad	— 67.31	—	9.58253
To Sine Comp. of ac	— 69.49	—	9.53797

CASE the Ninth.

Given two Angles, and a Side included to find either Side.

Given	$\left\{ \begin{array}{l} a - 27.19 \\ b - 131.52 \\ ab - 42.12 \end{array} \right\}$	Required bc	Plate 4. Fig. 25.
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Here by Solution the ninth, the Perpendicular may fall from a or b ; but in this Case let it fall from b , because it is the Concourse of the Sides given and required: Then to find the Vertical Angle abd , where ab is Middle Part, and a and b Extrems Conjunct; therefore,

As Tang. Comp. of a	— 27.19	—	10.28692
To Radius	—	—	90.00—10.00000
So Sine Comp. of ab	— 42.12	—	9.86970
To Tang. Comp. of abd	— 69.4	—	9.58278

From

From the whole Angle $a b c$ — 131.52

Subtract the Angle $a b d$ — 69. 4

Remains the Angle $c b d$ — 62.48

Then by Conclusion the eighth.

As Sine Comp. of $c b d$ — 62.48 Co ar 0.33999

To Sine Comp. of $a b d$ — 69. 4 — 9.55301

So Tang of $a b$ — 42.12 — 9.95748

To Tang. of $b c$ — 35.19 — 9.85048

CASE the Tenth.

Given two Sides, and an Angle included to, find the other Angle.

Plate 4.

Fig. 26.

Given $\begin{cases} a = 27.19 \\ b = 131.52 \\ c b = 42.12 \end{cases}$ Required $\angle$

Here by Solution the ninth, and the General Rule in Case the seventh, the Perpendicular must fall from b , upon $a c$; then find the Vertical Angle $a b d$, by the Proportion In Case the ninth, which is there found to be 69 4.

Then

Then by Conclusion the fifth.

As Sine of $a b d$ —————	69. 4	Co ar	0.02966
To Sine Comp. of a —————	27.19	— —	9.94865
So Sine of $c b d$ —————	62.48	— —	9.94910
To Sine Comp. of c —————	32.13	— —	9.92741

CASE the Eleventh.

Given three Sides, to find an Angle.

Given	$\left\{ \begin{array}{l} a b 42 12 \\ b c 35 20 \\ a c 69 48 \end{array} \right\}$	Required a	Plate 4. Fig. 27.
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In this Case the Perpendicular must be let fall from the Angle b , upon the Side $a c$, by Conclusion the twelfth, and the Rules following.

Then by Conclusion the twelfth.

As Tang. of $\frac{1}{2}$ the Base —	34.54	Co ar	0.15639
To Tang. of $\frac{1}{2}$ the Sum	} 38.46	— —	9.90475
of the Sides,			
So Tang. of $\frac{1}{2}$ Difference	} 3.26	— —	8.77811
of the Sides,			
To Tang. $\frac{1}{2}$ Diff. of Seg.	} 3.57	— —	8.83925
of the Base			

To

To the Half-base ————— 34.54

Add the Half-difference ————— 3.57

The Sum is the greater Segment ————— 38.51

Or the Difference is the lesser Segment — 30.57

Then by Axiom the first,

As Radius ————— 90.00 — 10.00000

To Tang. of a d ————— 38.51 — 9.90604

So Tang. Comp. of a b ————— 42.12 — 10.04251

To Sine Comp. of a ————— 27.20 — 9.94855

CASE the Twelfth.

Given three Angles, to find a Side.

Plate 4.
Fig. 28. Given $\left\{ \begin{array}{l} a - 27\ 19 \\ b - 131\ 52 \\ c - 32\ 12 \end{array} \right\}$ Required a b

NOTE, Before you can resolve this Case, you may turn the Angles into Sides, and the Sides into Angles; and then the Operation in the supplemental Triangle will be the same as in Case the eleventh; only in that Conversion, you must take the Complement of the greatest Angle to 180, as in Solution the seventh.

The

The Given-angles being thus turned into Sides, and the Contrary; the Operation is the same as if you had

$$\text{Given } \left\{ \begin{array}{l} ab = 27.19 \\ bc = 32.12 \\ ca = 48.8 \end{array} \right\} \text{Required } a \quad \text{Plate 4. Fig. 27.}$$

By Conclusion the twelfth, let fall the Perpendicular upon the greatest Side ac , from its opposite Angle b — then;

$$\begin{array}{l} \text{As Tang. } \frac{1}{2} \text{ the Base } ac = 24.4 \text{ Co ar } 0.35006 \\ \text{To Tang. half Sum of} \\ \text{the Sides } ab \text{ and } bc \left\{ \begin{array}{l} 29.45 \\ 2.26 \end{array} \right. \text{ — — } 9.75705 \\ \text{So Tang. half Differ.} \\ \text{of the said Sides} \left\{ \begin{array}{l} 2.26 \\ 2.26 \end{array} \right. \text{ — — } 8.62834 \end{array}$$

$$\begin{array}{l} \text{To Tang. half Differ.} \\ \text{Segm. of the Base} \left\{ \begin{array}{l} 3.7 \\ 3.7 \end{array} \right. \text{ — — } 8.73545 \end{array}$$

$$\text{Half Base} \text{ — — — — } 24.4$$

$$\text{Half Differ. Segm. of the Base } 3.7$$

$$\text{Their Sum is the greater Base } dc = 27.11$$

$$\text{Their Differ. is the lesser Base } ad = 20.57$$

Then by Axiom the first.

$$\text{As Radius} \text{ — — — — } 90.00 \text{ — } 10.00000$$

$$\text{To Tang. Comp. of } ab \text{ — — } 27.19 \text{ — } 10.28692$$

$$\text{So Tang. of } ad \text{ — — — — } 20.57 \text{ — } 9.58304$$

$$\text{To Sine Comp. of } a \text{ — — — — } 42.10 \text{ — } 9.86998$$

The

90 Spherical Trigonometry, &c.

The Angle a in this Triangle, being equal to the Side $a b$, viz. $42^{\circ} 10'$, is the Answer required.

The same Answer is produced by the Operation grounded upon Axiom the third, with only the Difference of a Minute or two, occasioned by the omitting half Minutes in taking half of an odd Number, as appears by the following Calculation ;

For the Side $a b$, the Operation is,

Angle a	27.19	} Adj. Ang. }	S Co. ar.	0.33828	
Comp. b	48. 8		S Co. ar.	0.12802	
Angle c	32.12				
Sum —	107:39	} $\frac{1}{2}$ Sum Si.	53.21	— 9.90699	
$\frac{1}{2}$ Sum —	53:49 $\frac{1}{2}$		Remaind.	21 37 $\frac{1}{2}$	— 9.56646
An. c sub.	32:12				
Rem. —	21.37 $\frac{1}{2}$	Sum of the 4 Log.	19.93975		
		$\frac{1}{2}$ Sum Sine	21- 6	— 9.96987	

21 6 doubled, is $42^{\circ} 12'$, the Side $a b$ required

See a further Improvement of this Case, in the last Problem but one of Astronomy.

CHAP.

h<

1

a<

h<

5

a<

h<

9

a<

h<

13

a<

17

h<

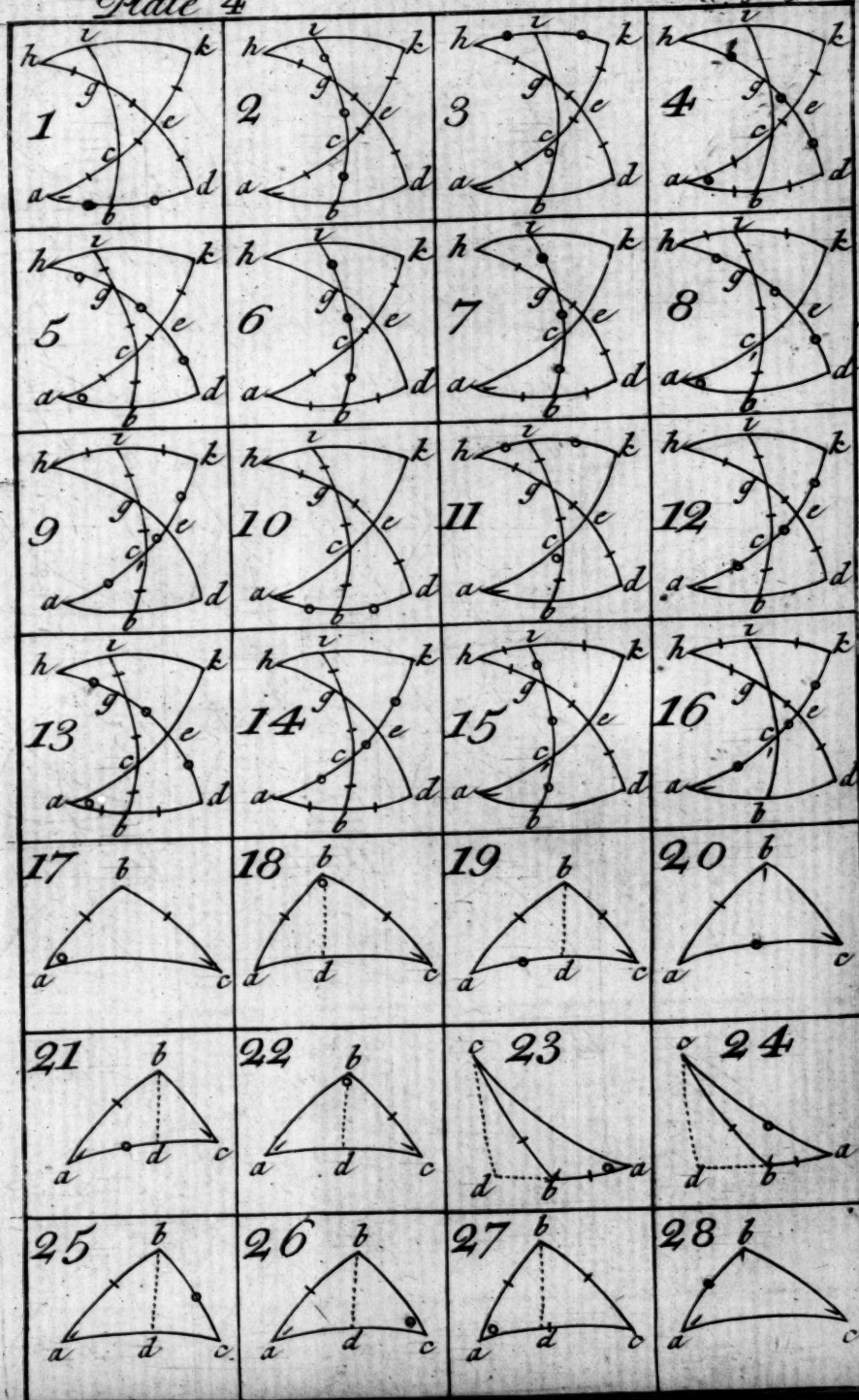
21

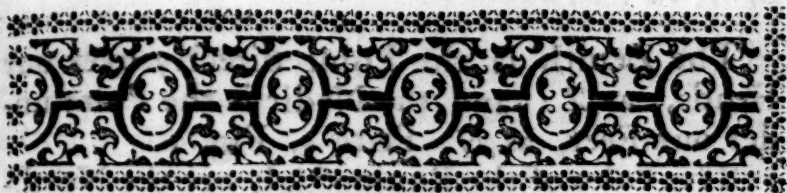
a<

h<

25

a<





CHAP. IV.

ASTRONOMY.



SECT. I.

Astronomical Definitions.



NOTE, Where Letters are refer'd to, and no Figure mention'd, the said Point, or Arch, *&c.* mark'd out by these Letters, may be found in all the 5 Projections; but where the Point, or Arch, cannot be seen in all, I shall refer to the Figure that best illustrates them: As Fig. 1. Fig. 2, *&c.* NOTE, All these Definitions refer to Plate 5.

POINTS.

P O I N T S.

The Zenith is the Point directly over our Heads, as *Z*; and the Nadir is the Point under our Feet, as *N*: Plate 5. Fig. 1.

The Poles of the Equinoctial, are the North-pole, as *P*, in all the Figures; and the South-pole, as *S*, Fig. 1. and 5. these are the Poles of the World — The Pole of any great Circle is every Way 90 Degrees distant from it: Thus the Zenith and Nadir are the Poles of the Horizon, &c.

Great CIRCLES.

The Meridians are great Circles passing through the Poles of the World, and cutting the Equinoctial at Right-angles: As the Primitive-circle, Fig. 1, and the Right-circle *n z s*, Fig. 2 — Of the Meridians, Two are called Colures, viz. The Solstitial-colure, as the Primitive-circle, Fig. 1. (which cuts the Ecliptick in Cancer and Capricorn) And the Equinoctial-colures, which cuts the Ecliptick in Aries and Libra: As the Right-circle *P L S*, Fig. 1, and *E P W*, Fig. 3.

The Equinoctial is a great Circle that divides the Northern and Southern Hemisphere, and cuts all Meridians at Right-angles: As *E L Q*, Fig. 1. *W Q E*, Fig. 2. The Primitive-circle Fig. 3, *E Q W*, Fig. 4, and *H Q O*, Fig. 5.

The

The Ecliptick makes an Angle of $23^{\circ} 30'$, with the Equinoctial in the first Point of Aries and Libra: As ELC , Fig. 1 — $W \odot E$, Fig. 2. and 3 — The Primitive, Fig. 4, and $HC \odot O$, Fig. 5. In it the twelve Signs are reckoned from West to East: As γ Aries, τ Taurus, Π Gemini, ♋ Cancer, Ω Leo, ♍ Virgo, ♎ Libra, ♏ Scorpio, ♐ Sagittarius, ♑ Capricorn, ♒ Aquarius, ♓ Pisces; each Sign containing 30 Degrees, makes in all 360 Degrees, as in all other great Circles.

The Horizon divides that part of the Sphere that is above the Earth, from that that is under the Earth: as HLO , Fig. 1 — The Primitive, Fig. 2, and HaO , Fig. 5.

The Azimuths are Circles intersecting each other in the Zenith and Nadir, and cuts the Horizon at Right-angles: As $Z \odot N$, Fig. 1 — $\odot ZR$, Fig. 4. And the Azimuth that cuts the Horizon in the East and West Points thereof, is called the Prime-vertical: As ZLN , Fig. 1, and WZE , Fig. 2. — And the Primitive, Fig. 5.

Circles of Longitude, cut the Ecliptick at Right-angles, and intersect each other in the Poles thereof: As EAW , and CAK , Fig. 4.

Lesser CIRCLES.

They are commonly Parallel to some great Circle, and hence called Parallels: As Parallels of Altitude, as $r \odot U$: and Parallels of Declination, as $g \odot m$, &c. Fig 1.

The

The Tropick of Cancer is a Parallel to the Equinoctial, distant 23d. 30m. Northward, and the Tropick of Capricorn the like Distance Southward. — The Artick Circle is distant from the Equinoctial Northwards, and the Antartick Southwards, 66d. 30m.

 Arches of lesser Circles are never Parts of Triangles.

Arches of great CIRCLES.

NOTE. In the following Definitions, what is said of the Sun, may be understood of a Star, or any Point in the Heavens.

—Sun's Altitude is an Arch of an Azimuth, contain'd between the Sun and the Horizon: As $\odot R$, Fig. 1, 2, 5.

Amplitude from the North or South, is an Arch of the Horizon, between the North or South Point of the Horizon, and the Point where the Sun Rises or Sets: As DO , Fig. 1.

Azimuth is an Arch of the Horizon, comprehended between the Point where the Azimuth Circle that the Sun is in, cuts the Horizon, and the North or South Points of the Horizon: Thus the Sun's Azimuth from the South, is represented by the Arch HR , Fig. 1.

—Right-ascension is an Arch of the Equator, contain'd between the Beginning of Aries (or in Astronomical operations, between the nearest

est Equinoctial-point) and that Point of the Equator, that comes to the Meridian with the Sun: As Lb , Fig. 1.

Oblique-ascension is an Arch of the Equinoctial, contain'd between the next Equinoctial Point, and that Point of the Equinoctial that Rises with the Sun, and is found by adding to, or subtracting from the Right-ascension; the

Ascensional Difference which is an Arch of the Equinoctial, contain'd between that Point of the Equinoctial that Rises with the Sun, and that that comes to the Meridian with him: As Ly , Fig. 1.

Longitude of the Sun or Stars, is an Arch of the Ecliptick, contain'd between the next Equinoctial Point, and the Sun; As $L\odot$, Fig. 1. — $E\odot$, Fig. 2 and 3; and $W\odot$ Fig. 4; and $O\odot$, Fig. 5.

Declination is an Arch of the Meridian, contain'd between the Equinoctial and the Sun; As $\odot b$, Fig. 1.

Latitude of a Star, is an Arch of a Circle of Longitude, comprehended between the Star and the Ecliptick: As $*C$, Fig. 4.

NOTE, The Sun always moves in the Ecliptick, and never hath any Latitude.

Longitude on the Earth, is measur'd by an Arch of the Equinoctial, contain'd between the Meridian, where the Longitude is suppos'd to begin, and the Meridian of the Place.

NOTE,

NOTE, Our *Engliſh* Geographers and Navigators of Late, do commonly begin their Longitude at *London*, and Number Eaſt or Weſt from thence.

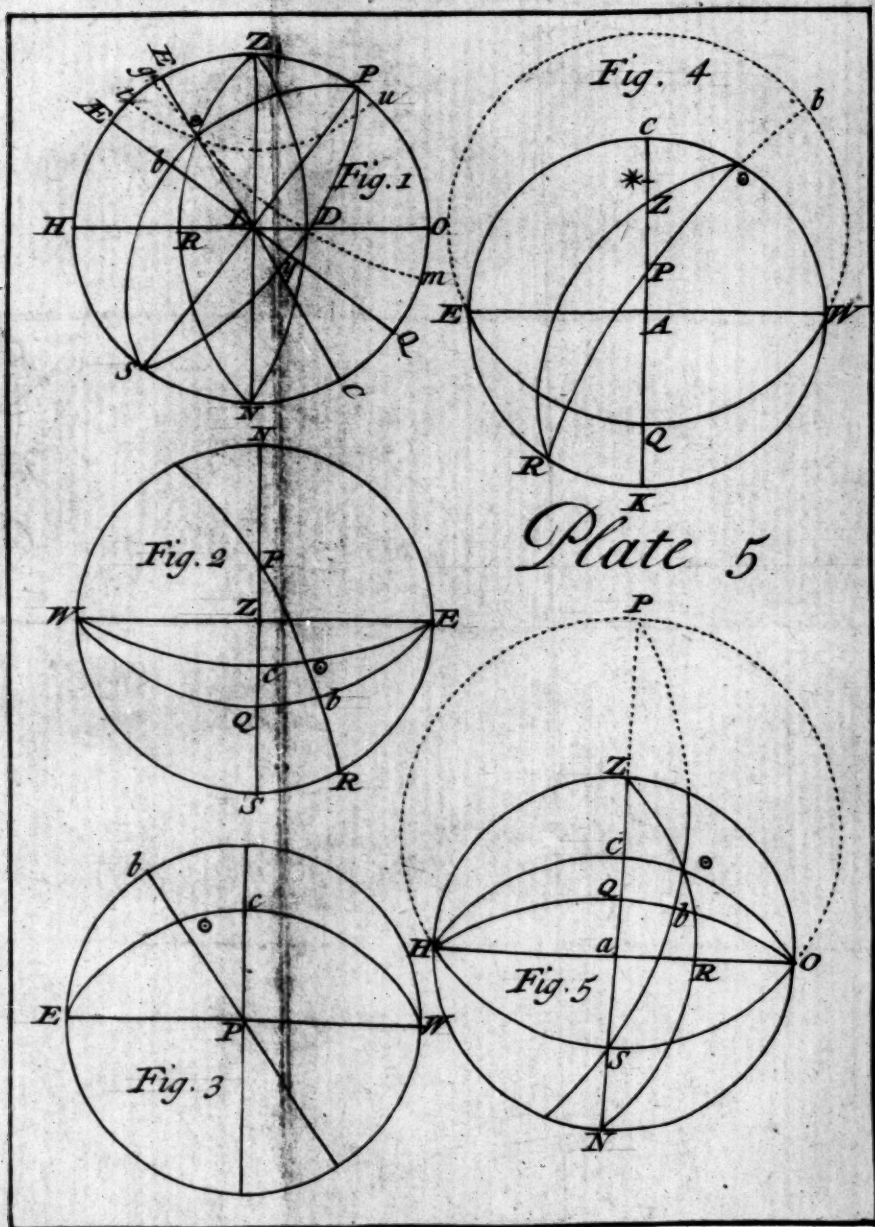
ANGLES.

The Hour of the Day is an Angle, made at the Pole, between the Meridian of the Sun, and the Meridian of the Place: Thus the Angle $Z P \odot$, is the Hour from Noon, when the Sun is any where in the Meridian $P \odot S$, Fig. 1.

The Sun's Azimuth is an Angle at the Zenith, contain'd between the Azimuth that the Sun is in, and the Prime-verticall (if reckon'd from the Eaſt or Weſt) or the Meridian, if reckon'd from the North or South: Thus the Angle $\odot Z L$, is the Sun's Azimuth from the Eaſt or Weſt; but $\odot Z E$, is the Sun's Azimuth from the South, Fig. 1.

The Sun's greateſt Declination is the Angle that the Ecliptick makes with the Equator: As the Angle $b L \odot$, Fig. 1.

NOTE, All theſe and other Angles in Astronomy, may alſo be reckon'd Arches of thoſe great Circles that are 90 Degrees diſtant from the Angular-point: Thus the Hour is an Arch of the Equinoctial: The Azimuth is an Arch of the Horizon, &c. Of which ſee more in the foregoing Definitions, and in Prob. 9. of Spherical Geometry.



S E C T. II.

Questions in ASTRONOMY.

Astronomy is no more but the Application of Spherical-trigonometry, or supposing the Arches of Circles, which constitute your given Triangle, to be Arches of the Horizon, Equator, Ecliptick, &c. and thus by truly knowing what Circles the Longitude, Declination, Right-ascension, &c. are Arches of, as is already taught at large in Chap. 4, Sect. 1, You have nothing to do in your Astronomical-operations, but what you are already taught in the Operations in Spherical-trigonometry; only to make the Work more intelligible, I shall instance in some of the most useful Questions in Astronomy; and for Variety, I shall perform the Operations by my Lord *Nepair's* Proportion, that the Learner may understand the Variety of Proportions, whereby Spherical-trigonometry and Astronomy may be performed.

NOTE, Whereas in most Cases in Astronomy, when you project them, you first draw the Primitive-circle, and cross it with two Diameters (as Plate 5, Fig. 1, the Primitive-circle *Z H N O*, is cross'd with two Diameters *Z N* and *H O*) therefore to avoid needless Repetitions, when you see at the first, *P C Q*, it signifies, Draw the Primitive-circle, and Quarter it, viz. Cross
F it

it with two Diameters, as above described, and then proceed as directed.

PROBLEM I.

Given { Sun's Longitude $60:0$
Greatest Declin. $23:30$ }

Required Sun's present Declination.

Plate 5. Geometrical Projection.

I shall Instance in this upon all the five Plains; and first, Upon the Plain of Fig 1. the Meridian.

NOTE, Although the Solution of this Problem has no regard to a given Latitude; yet because it seems more natural to suppose a given Latitude, I have instanced in Latitude $54:40$ North.

$P C Q$, and from O set off the Lat. $54:40$ to P , and likewise from H to S , and draw $P S$, the Axis of the World; and at Right-angles it, draw $\mathcal{A} Q$ the Equator; and at an Angle of $23:30$ to that, draw $E C$ the Ecliptick, upon which (because Longitude is an Arch of the Ecliptick) set off $\odot$'s Longitude 60 , from L to $\odot$, and draw the Meridian $P, \odot, S$; then (because the Declination is an Arch of the Meridian) the Arch $L, \odot$ measured by Prob. 8. Var. 3. is the Declination required.

2. Upon the Plain of the Horizon, *Fig. 2.*

$P C Q$, then is the Center Z the Zenith, and P the Pole; then 90 Degrees from P , draw the Equinoctial $W Q E$, and 23 30, from that the Ecliptick $W C E$; set off 60 Degrees the $\odot$'s Longitude from E to $\odot$, and draw the Meridian $P \odot b R$, the Arch $\odot b$ is the Declination required.

3. Upon the Plain of the Equator. *Fig. 3.*

$P C Q$, then is the Primitive Circle the Equator, and P the Pole; draw $E C W$ the Ecliptick, to make an Angle of 23 30, with the Primitive at E and w , and set off $\odot$'s Longitude 60, 0 from E to $\odot$, and draw the Meridian $b \odot P$; then by $\odot$ measured by Prob. 8. Var. 2. is the $\odot$'s Declination required.

4. Upon the Plain of the Ecliptick *Fig. 4.*

$P C Q$, then 23 30 from the Ecliptick, as through Q , draw the Equator $E Q W$ continued round; because the Triangle falls without the Primitive-circle; and having found P the Pole of the World, which is known to be 23 30 distant from A the Pole of the Equator, draw the Meridian $R P \odot b$, the Arch $\odot b$ is the Declination required.

NOTE, Any Arch without the Primitive-circle, is measured by the same Rules in Prob. 8. that an Arch within the Primitive-circle is

measured; and here I have contrived the Projected-triangle to fall without, to shew the Reader the Variety.

5. Upon the Plain of the Prime-vertical Fig. 5.

$P C Q$, then is Z the Zenith, draw $H C O$ the Ecliptick, and $H Q O$ the Equator; set off $\odot$'s Longitude 60 , o from O to $\odot$, and draw the Meridian $P, \odot, S$, to cut the Equator in b ; the Arch $b \odot$ is the Declination required.

NOTE, The Triangle $o b \odot$ falling within the Primitive, all that Part falling without, express'd by prick'd Lines, might be omitted; only here I have inserted it to let the Learner see the Reason of his Work.

Trigonometrical Calculations.

The Triangle $L b \odot$, Fig. 1, or $E b \odot$, Fig. 2, 3, and $w b \odot$, Fig. 4, or $O b \odot$, Fig. 5, is the Triangle here proposed, in which you have given $L \odot$, Fig. 1, or $E \odot$, Fig. 2, 3, or $w \odot$, Fig. 4, or $O \odot$, Fig. 5, the Sun's Longitude 60 o ; and the Angle L , Fig. 1, or E , Fig. 2 and 3, w , Fig. 4, or O , Fig. 5, to find the Arch $b \odot$, by Case the second.

Here the Sun's Declination-required is Middle-part, and the other two, Extrems-disjunct; therefore,

As Radius ————— 90: 0:10.00000
 To Sine of Greatest-declination 23:30 9:60070
 So Sine of Sun's Longitude ——— 60:00. 9.93753
 To Sine of Present-declin. req. 20 12 9.53823

P R O B L E M II.

Given { ☉'s Longitude 60 0 } Req. ☉ Rig. Asc.
 ☉ Greatest-de. 23 30

'Tis projected as Prob. 1. being the same Data.

The Given angle is Middle-part, and the other Extreame-conjunct — Therefore by Case the first.

As Tang. Comp. ☉ Long. ——— 60 0. 9.76144
 To Radius ————— 90 0 10.00000
 So Sine-com. Greatest-dec. S C. 23 30. 9.96240
 To Tang. of ☉ Right-ascens. 57:48:10.20096

P R O B L E M III.

Given { ☉ Greatest-declin. } Req. ☉'s Long.
 ☉ Present-declin.

Greatest-declination 23 30 Present-declination 20 12 *Plate 5. Fig. 1.*

To project it upon the Plain of the Meridian.

$P C Q$, and draw $P S$; also draw $E Q$, the Equator, and $E C$, the Ecliptick, &c. as in Prob. 1. of Astronomy; then draw the Parallel of Declination $g \odot m$, and where it cuts the Ecliptick, as in the Point $\odot$, draw the Meridian $P \odot S$; then is $L \odot$ the Longitude required by Case the twelfth.

As Sine-ext. Disj. greatest Dec. 23.30. 9.60070
To Radius ————— 90 00.10.00000
So Sine Mid. part present Decl. 20 12. 9.53819

To Sine-ext. Disj. $\odot$ Longit. — 60 0 9.93749.

P R O B L E M IV.

{ Given $\odot$'s Declination — 20 12 }
{ Greatest Declination — 23.30 }

Plate 5. Required $\odot$ Right-ascension

Fig. 1.

Projected as Prob. 3. — Then by Case the tenth

As Radius ————— 90 0.10.00000
To Tangent-compl. Extream- }
conj. greatest Declin. $T C$ } 23 30.10.36170
So Tang. Ext. Con. present Dec. 20 12. 9.56576

To Sine Mid. part Right-ascen. 57:48 9.92746
P R O-

PROBLEM V.

Plate 6.

Fig. 1.

Given { ☉ greatest Dec. 23.30 } Req. ☉'s Lon.
 Right-ascension 57.48 }

PCQ, and as you have no respect to Latitude in this, as well as all the foregoing Problems, you may let your Perpendicular Diameter represent the Axis of the World, and the other the Equator; then at an Angle of 23 30, with the Equator, draw *EC* the Ecliptick; then upon the Equator, set off the Right-ascension (because the Right-ascension is always an Arch of the Equator) from *L* to *b*, and draw the Meridian *PbS*, and observe where it cuts the Ecliptick: As in the Point ☉, the Arch *L☉*, is the ☉'s Longitude required. Then by Case the ninth.

As Tang. Ex. Conj. Right-asc. 57:48:10.20084

To Radius ————— 90:00.10.00000

So Sine Comp. Mid. pt. gr. Dec. 23:30. 9.96240

To Tan. Com. Ex. Conj. Long. 60: 0 9.76156

PROBLEM VI.

Given { ☉'s greatest Dec. 23.30 } Req. ☉ Decl.
 Right-ascension - 57.48 }

Projected as Prob. 5 — Then by Case the Seventh

As Tan. Com. Ex. Con. Gr. De. 23.30 10.36170
 To Radius ————— 90 00.10.00000
 So Sine Mid. Pt. Right-ascen. 57.48. 9:92747
 To Tang. Ex. Conj. ☉ Declin. 20:12 9.56577

P R O B L E M VII.

Plate 6.
 Fig. 1. Given { Lat. of the Place 54 40 } Req.
 ☉ Declination - 20 12

Ascensional-difference ————— Prob. 7
 ☉ Rise and Set ————— Prob. 8
 Length of Day or Night ————— Prob. 9
 ☉'s Amplitude ————— Prob. 10
 What Hour ☉ is East ————— Prob. 11
 ☉'s Altitude when East ————— Prob. 12
 ☉ Altitude at 6 ————— Prob. 13
 ☉'s Azimuth at 6 ————— Prob. 14

P C Q, set off the Latitude 54 40, from *O* to *P*, and from *H* to *S*, and draw the Axis *P, S*, and at Right-angles to it, draw the Equator, *Æ Q*, and at an Angle of 23 30, draw *E C* the Ecliptick; draw the Parallel of Declination *G ☉ M*, distant from the Equator 20 12, equal to the Given-declination; and where it cuts the Ecliptick, as in ☉, draw the Meridian *p ☉ S*, and the Azimuth *Z ☉ N*; also where the Parallel cuts the Horizon, as in *D*, draw the Meridian *p D S*; and where the Parallel cuts the Prime-vertical, as in 3, draw the

the Meridian $P 3 S$; also where the Parallel cuts the Axis $P S$, as in 5, draw the Azimuth $Z 5 N$; which done, the Arch $L y$ is the Ascensional-difference, $L D$ the Amplitude from the East or West, because $G M$ is the Parallel the Sun moves in that Day, and the Point D is the Point of Sun's Rising: $D O$ the Amplitude from the North, the Angle $3 P L$ equal to the Arch $2 L$, is the Hour from 6 that the Sun is East or West, viz. In the Point 3, the Point 3 being the Point that the Sun is in when East or West; and hence the Arch $3 L$ is the Sun's Altitude when East or West, and the Arch $5 6$, is the Sun's Altitude at 6, and the Arch $L 6$, is the Sun's Azimuth at 6, taken from the East or West, &c. Then for Trigonometrical-operation.

First, For the Ascensional-difference.

In the Triangle $L y D$, you have given the Angle at L the Comp. Latitude, and the Leg $D y$ the Declination to find $L y$ the Ascensional Difference: Therefore by Case the Tenth.

As Radius	_____	90	0.10.00000
To Tang. of Latitude	_____	54	40.10.14941
So Tang. of Declination	_____	20	12. 9.56576
To Sine of Ascensional-differ.	_____	31.16.	9.71517

PROBLEM VIII.

To find the Time of Sun's Rising and Setting.

The Ascensional Difference $31^{\circ} 16'$ converted into Time, allowing 15° Degrees to one Hour, and one Degree to 4 Minutes of Time, gives the Hours and Minutes that the Sun Rises before 6 in Summer, or after 6 in Winter: Thus $31^{\circ} 16'$, gives 2 Hours 5 Minutes, which subtracted from 6, leave 3 Hours 55 Minutes in the Morning for Sun's Rising; or added to 6, makes 8 Hours 5 Minutes at Night for Sun's Setting, &c. and the contrary in South Declination, when the Sun Rises after, and Sets before 6; and then, instead of adding, you must subtract, and the contrary —

PROBLEM IX.

To find the Length of the Day or Night.

The Sun's Rising found by Prob. 8. being doubled, gives the Length of the Night, and double Sun's Setting, gives the Length of the Day: And thus, as by Prob. 8. you find Sun Sets at 8 Hours 5 Minutes, which doubled, gives 16 Hours 10 Minutes for the Length of the Day; and the Sun's Rising, viz. 3 Hours 55 Minutes doubled, gives the Length of the Night,

Night, which so found, is 7 Hours 50 Minutes, &c. —

PROBLEM X.

Plate 6.

Fig. 1.

Latitude and Declination given, to find the Amplitude.

In the former Projection from the same Data, you have given in the Triangle LYD , the Angle at L the Complement of Latitude, and the Leg Dy the Declination, to find the Hypotenuse LD the Amplitude; and here the Declination Dy is Middle Part, and the other Extream Disjuncts; therefore by Case 12/

As Sine Comp. Latitude, — $SC\ 54\ 40\ 9.76218$
 To Radius; ————— $90\ 00\ 10.00000$
 So Sine of Declination, ————— $20\ 12\ 9.53819$

To Sine of Amplitude. ————— $36\ 40\ 9.77601$

This is the same Proportion that arises from the Globical Projection.

PROBLEM XI.

Latitude of the Place $54\ 40$, and Sun's Declination $20:12$, given, to find what Hour and Minute the Sun will be due East or West.

Projected as in Prob. 7, being the same Data.

F 6

This

This falls in the Triangle $L\ 2\ 3$, in which you have given the Angle $2\ L\ 3$, the Latitude $54\ 40$, and the Leg $2\ 3$, the Declination to find the Leg $2\ L$, the Hour from 6 when the Sun is in the Point 3, viz. Due East or West, and falls in Case the Tenth of Right-angled Spherical Triangles; therefore,

As Radius, ————— 90 0:10.00000
 To Tangent Compl. Latitude, $54\ 40$. 9.85059
 So Tangent Declination, ——— $20\ 12$. 9.56576

To Sine of the Hour from 6. 15. 7 9.41635

Which gives one Hour to be added to 6 in the Morning, or subtracted from 6 in the Afternoon, the Sum 7 is the Time when the Sun is East, and the Remainder 5 the Time when he is due West, &c. ———

Plate 6. PROBLEM XII.

Fig. 1.

Having the Latitude $54\ 40$, and Declination $20\ 12$, to find the Sun's Altitude when East or West.

This falls also in the Triangle $L\ 2\ 3$, the Angle L and Leg $2\ 3$, given to find the Hypotenuse $L\ 3$, by Case 12.

As Sine of Latitude, ———— 54 40 9.91158
 To Radius; ————— 90:00.10.00000
 So Sine Declination, ———— 20:12. 9.53819
 To Si. of ☉ Alt. when E. or W. 25: 3: 9.62661

P R O B L E M XIII. *Plate 6.*
Fig. 1.

By the Latitude 54 40, and Declination 20 12, to find the Sun's Altitude at 6.

This projected as in Prob. 7, falls in the Triangle *L 5 6*, in which is given the Angle *L*, the Latitude, 54 40, and *L 5*, the Declination, 20 12, to find the Side *5 6*, the Point *5*, being the Point the Sun is in at 6, and the Arch *5 6* his Altitude above the Horizon at that Time; therefore by Case 2.

As Radius, ———— 90 0.10.00000
 To Sine Latitude; ———— 54:40. 9.91158
 So Sine Declination, ———— 20 12 9.53819
 To Sine of Altitude at 6. ———— 16:21. 9.44977

P R O B L E M XIV. *Plate 6.*
Fig. 1.

Given the Latitude 54 40, and Declination 20 12, to find Sun's Azimuth at 6, which falls in the same Triangle *L 5 6*, in which is given, as in Prob. 13, to find the Leg *L 6*, by Case the First.

As

As Tang. Comp. Decl. — T C 20:12:10.43424
 To Radius; ————— 90 00.10.00000
 So Sine Comp. Latitude — S C 54 40. 9.76218
 To Tang. Sun's Azimuth at 6. 12. 1 9.32794

Plate 6. PROBLEM XV.

Fig. 1.

Latitude and Ascensional Difference given, to find Sun's Declination.

This; and the following Problems, are only other Parts given and required in the fore-mentioned Triangles, as the Learner may easily see by what is already laid down in the fore-going Problems; and therefore I shall not need to enlarge, but shall lay down the Questions, and leave the Operations for the Reader's Practice.

In the Triangle $L y d$, there is given $L y$, the Ascensional Difference 31 16, and the Angle L the Complement of Latitude 35 20, to find $D y$, the Declination, by Case the Seventh.

As Tang. Comp. of L ,
 To Radius;
 So Sine of $L y$,
 To Tang. of $D y$.

P R O.

PROBLEM XVI. *Plate 6.**Fig. 1.*

Given the Latitude of the Place $54^{\circ} 40'$, and the Ascensional Difference $31^{\circ} 16'$, to find the Sun's Amplitude.

In the Triangle $L y D$, there is given the Angle at L , and the Leg adjacent Ly , to find the Hypothenuse LD , by Case the Ninth.

As Tangent Ly ,

To Radius;

So Sine of Complement of L ,

To Tangent Complement of $L d$.

PROBLEM XVII. *Plate 6.**Fig. 1.*

Given the Latitude of the Place $54^{\circ} 40'$, and the Amplitude of the Sun $36^{\circ} 40'$, to find the Sun's Declination.

In the Triangle $L y D$, is given the Angle at L the Complement of Latitude, and LD the Amplitude to find the Declination $D y$, by Case the Second.

As Tangent Complement of L ,

To Radius;

So Sine Complement of $D L$,

To Tangent of $D y$.

P R O.

Plate 6. PROBLEM XVII.

Fig. 1.

Given the Latitude $54^{\circ} 40'$, and Sun's Amplitude $36^{\circ} 40'$, to find the Ascensional Difference.

In the Triangle $L y d$, is given the Angle L , and the Hypothenuse $L D$, to find $L y$ by Case the First.

As Tangent Complement of $L D$,
To Radius;
So Sine Complement of L ,
To Tangent $L y$.

Plate 6. PROBLEM XIX.

Fig. 1.

The Sun's Declination $20^{\circ} 12'$, and Ascensional Difference $31^{\circ} 16'$, given to find the Latitude of the Place.

In the Triangle $L y d$, is given $L y$, and $D y$, to find the Angle L , by Case the Thirtieth.

As Tangent $D y$,
To Radius;
So Sine of $L y$,
To Tangent Complement of L .

PROBLEM XX. *Plate 6.
Fig. 1.*

Given as in Prob. 19, to find Sun's Amplitude $L d$, by Case the Fourteenth.

As Radius,
To Sine Complement $L y$;
So Sine Complement $D y$,
To Sine Complement $L D$.

I might advance to a great Number of Problems, which may be deduced from the several Right-angled Triangles, contain'd in this Figure; but not willing needlessly to encrease the Number, I shall confine my self only to those where the Latitude is required, they being of most Use in Practice at Sea, the Artist being commonly fitted with Books, Instruments, &c. whereby the Sun's Altitude Declination, &c. is attain'd without Trigonometrical Calculation; and by the Help of these Problems following, the Latitude may sometimes be attain'd in a Morning, or Evening, when an Observation cannot be had at Noon.

Plate 6. PROBLEM XXI.

Fig. 1.

Given the Sun's Declination $20^{\circ} 12'$ North, and the Amplitude $36^{\circ} 40'$, to find the Latitude of the Place.

In the Triangle $L y D$, is given the Hypotenuse $L D$, and the Leg $L y$, to find the Angle L , by Case the Fifth.

As Sine $L D$,
To Radius;
So Sine of $D y$,
To Sine of L .

Plate 6. PROBLEM XXII.

Fig. 1.

The Sun's Amplitude $36^{\circ} 40'$, and Ascensional Difference $31^{\circ} 16'$, given, to find the Latitude of the Place.

In the Triangle $L y D$, you have given the Hypotenuse $L D$, and the Leg $L y$, to find the Angle L , by Case the Fourth.

As

As Radius,
To Tangent $L y$;
So Tangent Complement of $L D$,
To Sine Complement of L .

PROBLEM XXIII. *Plate 6.*
Fig. 1.

Given the Sun's Declination $20^{\circ} 12'$, and his Altitude at Six a Clock $16^{\circ} 22'$, to find the Latitude.

In the Triangle $L 5 6$, is given the Hypotenuse $L 5$ the Declination, and the Leg $5 6$ the Altitude at 6, to find the Angle L the Latitude, by Case the Fifth.

As Sine of $L 5$,
To Radius;
So Sine of $5 6$,
To Sine of the Angle at L .

PROBLEM XXIV. *Plate 6.*
Fig. 1.

Given the Sun's Declination $20^{\circ} 12'$, his Azimuth at 6 $12^{\circ} 1'$, to find the Latitude.

In the Triangle $L 5 6$, is given $L 5$ the Declination, and $L 6$ the Azimuth at 6, to find the Angle at L , by Case the Fourth.

As

As Radius,
 To Tangent $L\ 6$;
 So Tangent Complement of $L\ 5$,
 To Sine Complement of L required.

Or in the Quadrantal Triangle $L\ Z\ 5$, is given $L\ 5$ the Sun's Declination $20\ 12$, and the Angle Z the Sun's Azimuth at 6 , viz. $12\ 1$, to find the Angle at L , the Complement of Latitude required, which by the Rules given Chap 3, for Quadrantal Triangles, falls under the following Proportion.

As Radius, ————— 10.00000
 To Tang. of the Ang. Z ; — $12. 1$ — 9.32809
 So Tang. Com. Dec. $L\ 5$ — $20\ 12$ — 20.43423
 To Sine Com. Lat. $Z\ L\ 5$ — $54\ 39$ — 9.76232

Plate 6. PROBLEM XXV.

Fig. 1.

Given the Sun's Declination, together with his Altitude, taken due East or West, to find the Latitude.

In the Triangle $L\ 5\ 6$, is given the Declination $L\ 5$, $20\ 12$, and the Sun's Altitude, when East or West, $25\ 3$, the Leg $5\ 6$, to find the Angle L , the Complement of Latitude, by Case the Fifth.

As

As Sine of L 5 the Declination,
To Radius;
So Sine of 56, the Sun's Altitude, when East
or West,
To the Sine Complement of L the Latitude
required.

Or in the Quadrantal Triangle, Z L 5, is
given Z 5, the Complement of Altitude, when
East or West; and L 5, the Declination, to
find the Angle L , the Complement of La-
titude.

As Sine of L 5 the Declination,
To Radius;
So the Sine Complement of Z 5 (which is the
Sine of 56 the Sun's Altitude,)
To the Sine Complement of L (which is the
Sine of Latitude) required.



S E C T. III.

Astronomical PROBLEMS whose Solutions depend upon Oblique Spherical Triangles.

Plate 6.

P R O B L E M I.

Fig. 2.

GIVEN the Sun's Altitude Declination, and Azimuth, to find the Hour of the Day.

E X A M P L E.

The Sun's Altitude, — 47 48 }
 The Sun's Dec. North, 20 12 } Req. the Hour.
 Azimuth from the Sou. 48 8 }

I have given this the 6th Place, because it falls under the first Case of Oblique Spherical Triangles; my Design being (for the Learner's better Instruction) to instance one Question in each Case, and to project each upon the Plain of such a great Circle, that it may come under this,

General

General Rule for projecting Oblique Spherical Triangles.

When there is given two Sides, and an Angle make a given Side upon the Primitive, and the given Angle at the Primitive; but if two Angles and a Side make both the Angles at the Primitive, and let the given Side fall within, except it be the Side contain'd between the given Angles; but when only three Sides is given, as in Case 11, or three Angles, as in Case 12, you may project upon the Plain of any of the three Circles, whose Arches compose the Triangle.

NOTE, You may in many Cases project the Triangle at the Center of the Primitive, as in Prob. 7 and 12 following.

It is most convenient to project this Problem upon the Plain of an Azimuth, because the Sun's Altitude (which is an Arch of an Azimuth) is given, and the Sun's Azimuth is an Angle adjacent to that given Side; therefore,

PCQ , and set the Sun's Altitude $47^{\circ} 48'$, upon the Primitive from H to $\odot$, and draw the Meridian of the Place $\propto Pn$, to make an Angle $131^{\circ} 52'$ with the Primitive (equal to the Sun's Azimuth from the North, or the Complement of the Azimuth from the South) by Prob. 3, Case 2; and because the Sun is distant from the Pole $69^{\circ} 48'$, equal to the Complement of Declination, draw the Parallel aPb , at the said

said Distance from the Sun, by Prob. 7, Case 3; and because the Pole is somewhere in that Parallel, and also in the Meridian of the Place, it must be where they cross each other, as in P ; then through $\odot$ and P , draw the Meridian $\odot P C$, to cut the Primitive in the opposite Points $\odot$ and C , and the Triangle is finished; and in the Triangle $Z P \odot$, there is given $Z \odot$, the Complement of Altitude $42\ 12$, and $\odot P$, the Complement of Declination $69\ 48$, and the Angle $\odot Z P$, the Azimuth from the South, to find the Angle $Z P \odot$, the Hour from Noon, by Case the First.

As Sine of $\odot P$, ———	69 48	Co ar	0.02757
To Sine of $\odot Z P$; ———	131 52	— —	9.87198
So Sine of $\odot Z$, ———	42 12	— —	9.82718
To Sine of $Z P \odot$. — —	32 12	— —	9.72673

This $32\ 12$, reduced to Time, allowing 15 Degrees to an Hour, and every Degree 4 Minutes, gives 2 Hours 9 Minutes, viz. 15 Minutes after 9 in the Morning, or 9 Minutes past 2 in the Afternoon.

Plate 6. PROBLEM II.

Fig. 3.

Given the Latitude of the Place, with the Sun's Altitude, and Hour of the Day, to find the Azimuth.

Latitude

Latitude North, ———— 44 40 }
 Sun's Altitude, ———— 47 48 } to find the
 Ho. from No. 2 Ho. 9 M. or 32 12 } Azimuth.

This must be projected upon the Plain of the Meridian, because the Latitude (an Arch of the Meridian) and the Hour (an Angle adjacent) is given; therefore,

PCQ , let the Latitude 54 40, from O to P , or its Complement from Z to P , draw $P \odot S$, to make the given Angle at P 32 12, by Prob. 3, Case 2, and draw the given Parallel of Altitude 47 48 above the Horizon, by Prob. 7, Case 2, as $b \odot c$; and where that cuts the Meridian $P \odot S$, as in the Point $\odot$ draw the Azimuth $Z \odot N$, and the Triangle $Z P \odot$ is finished, in which you have given,

The Comp. Lat. $Z P$, }
 The Comp. Alt. $Z \odot$, } To find the Azimuth,
 The Angle $Z P \odot$, — } $\odot Z P$ by Case the Sec.

The Triangle $Z P \odot$, must first be supposed to be divided into two Right-angled Triangles, by the Directions given in Case the Second, of Oblique Spherical Triangles; which done, the Operation is,

As Tang. Comp. of $Z P \odot$, — 32 12 10.20084
 To Radius; ————— 10.00000
 So Sine Comp. of $Z P$, ——— 35.20 9.91158
 To Tan. Com. of $P Z \odot$, ——— 62 48 9.71074
 G Then

Then by Conclusion the Eighth,

As Tangent of $\odot Z$, ——— 42.12 Co ar 0.04251
 To Sine Comp. $P Z e$; — 62 48 ——— 9.66001
 So Tang. of $Z P$, ——— 35 20 ——— 9.85059

To Sine Comp. $\odot Z e$. — 69. 4 ——— 9.55311

To the Angle $P Z e$, 62 48

Add the Ang. $\odot Z e$, 69 4

The Sum is $P Z \odot$. 131.52 the Azimuth requi.

The Reason of this Conclusion, made use of in the last Operation of this Problem, and also of the rest mentioned under Axiom the Second, of Spherical Trigonometry, may be deduced from these Considerations, viz.

Suppose in the Triangle $Z P \odot$, before-mentioned, the Perpendicular $Z e$, which divides it into two Right-angled Triangles, were given; and $\odot Z$, and $Z P$, which are now become Hypothenufes, to the two Right-angle Triangles, were also given, to find the Vertical Angles, by Case the Fourth, the Proportion would be,

As Tangent Complement $Z e$,
 To Radius;

So Tangent Complement $Z P$,
 To Sine Complement $P Z e$.

As Tangent Complement $Z e$,
 To Radius;

So Tangent Complement $Z \odot$,
 To Sine Complement $\odot Z E$.

The

The two first Terms in both Proportions being the same, take them both away, and the rest will retain the same Proportion (*Eucl. l. 5. p. 11.*) viz.

As Tangent Complement $Z P$,
 To Sine Complement $P Z E$;
 So Tangent Complement $Z \odot$,
 To Sine Complement $\odot Z e$.

And if the Tangent Complements are directly, the Tangents must be mutually, or reciprocally proportionable to the Sine Complements of their respective Angles; and by this Way of Reckoning, you may find the Ground of all the rest of the Conclusions, always imagining the Perpendicular, which is common to both Triangles, to be given; and the other Parts given, to be the same in both Triangles, whether the Hypothenuse, or Angle, at the Base, &c. And by this you may form two Proportions, whose two first Terms will be the same, which being destroyed, the other retaining the same Proportion, is the Conclusion required.

P R O B L E M III.

Given the Latitude of the Place, with the Sun's Altitude, and the Hour of the Day, to find the Declination.

G 2

Latitude

Latitude North, - 54 40 }
 Sun's Altitude, — 47 48 } to find the Declinat,
 Hour from Noon, 32 12 } $\odot P$, by Case the Sec.

Plate 6. Projected as Prob. 2.

Fig. 3.

Which being done, and the Perpendicular $Z e$ let fall by Solution the Ninth; the Operation is,

As Tang. Comp. $Z P$, — — — 35 20 10.14948
 To Radius; — — — — — 10.00000
 So Sine Comp. $Z P e$, — — — 32 12 9.92747
 To Tangent $P e$. — — — 30 57 9.77799

Then by Conclusion the Sixth.

As Sine Comp. $Z P$, — — 35 20 Co ar 0.08841
 To Sine Comp. $e P$; — — 30 57 — — 9.93329
 So Sine Comp. $\odot Z$, — — 42 12 — — 9.86970
 To Sine Comp. $\odot e$. — — 38 51 — — 9.89149

To $P e$, — — 30 57

Add $\odot e$, — 38 51

Sum is $\odot P$, 69.48 the Complement of Declination, which taken from 90, leaves 20 12, the Declination required.

For

For other Methods of Solution, see Case the Third, of Oblique Spherical Triangles.

PROBLEM IV. *Plate 6.*

Fig. 3.

Given the Sun's Altitude, Azimuth, and Hour from Noon, to find the Declination.

Altitude $H \odot$, — 47.48
 Azimuth $\odot Z P$, 131 52
 Hour $\odot P Z$, — 32 12 } to find the Dec. $\odot P$,
 by Case the Fourth.

Projection upon the Plain of the Meridian $P C Q$, draw the Azimuth $Z \odot N$, to make with the Primitive the given Angle $\odot Z P$, 131:52; draw the Parallel $b \odot c$, distant from Z , equal to the Complement of Altitude 42 12; and where that cuts the Azimuth $Z \odot N$, as in $\odot$, is the Point the Sun is in; therefore through the Point $\odot$, by Prob. 4, of Spherical Geometry, draw the Meridian $p \odot S$, to make the given Angle 32 12, with the Primitive at P , and then the Triangle is finished.

Now in the Triangle $Z P \odot$, is given $Z \odot$, the Complement of Altitude, $\odot Z P$ the Azimuth, and $Z P \odot$, the Hour to find $\odot P$, the Complement of Declination, by Case the Fourth, of Oblique Spherical Triangles, where the Case is proposed in the same Numbers; and therefore I shall not make any Repetition,

tition, but refer the Learner to that Case; the Complement of Declination is $69^{\circ} 50'$, and consequently the Declination $20^{\circ} 10'$.

Plate 6.

PROBLEM V.

Fig. 3.

Given the Sun's Altitude, Azimuth, and Hour of the Day, to find the Latitude.

Altitude, $-47^{\circ} 48'$

Azimuth, $131^{\circ} 52'$

Hour, $-32^{\circ} 12'$

} to find the Latitude by Case the Fifth.

The Data and Projection is the same, as in Prob. 4; but by Solution the Ninth, the Perpendicular falls without, as in Plate 6, Fig. 4; then in the Right-angled Triangle, $Z D \odot$, Right-angled at D , there is given the Hypotenuse, $Z \odot$, $42^{\circ} 12'$, and the Angle, $\odot Z D$, (the Complement of $P Z \odot$, to 180) viz. $48^{\circ} 8'$, to find $Z D$.

As Tang. Comp. $Z \odot$, $42^{\circ} 12'$ 10.04251

To Radius; 10.00000

So Sine Comp. $\odot Z D$, $48^{\circ} 8'$ 9.82436

To Tang. $Z D$, $31^{\circ} 11'$ 9.78187

Then

Then by Conclusion the Seventh.

As Tang. — $\odot$ $P D$, 32 12 $\odot$ ar 0 20 084
 To Sine $Z D$; — — — 31 11 — — — 9.71414
 So Tang. $\odot$ $Z D$, — — — 48 8 — — — 10.04759
 To Sine $P D$. — — — 66 33 — — — 9.96257

From $P D$, 66 33

Take $Z D$, 31 11

Rems $P Z$, 35 22 the Complement of Latitude;
 hence the Latitude required is 54 38.

PROBLEM VI.

Plate 6.

Fig. 3.

Given the Sun's Altitude, Azimuth, and Hour of the Day, to find the Angle of Position.

Altitude, 47 48
 Azimuth, 131 52
 Hour, — 32 32 } to find the Angle of Position,
 viz. the An. $Z \odot P$, by Ca. fix.

Projected as in Prob. 4, the Perpendicular falls without from $\odot$, as in Prob. 5. (See Plate 6, Fig. 4.) then in the Right-angled Triangle $Z D \odot$, is given, as in Prob. 4, to find the Angle $Z \odot D$.

As Tang. Comp. $\odot$ Z D, —	48 8	9.95240
To Radius; —		10.00000
So Sine Comp. $\odot$ Z —	42 12	9.86970
To Tang. Comp. Z $\odot$ D. —	50 25	9.91730

Then by Conclusion the Fifth.

As Sine Comp. $\odot$ Z D, —	48 8	Co ar	0.17561
To Sine of Z $\odot$ D; —	50 25	—	9.88688
So Sine Comp. $\odot$ P D, —	32 12	—	9.92746
To Sine of P $\odot$ D. —	77 43	—	9.98995

From the whole Angle P $\odot$ D, 77 43

Take the Angle Z $\odot$ D, — 50 25

Rest the Angle P $\odot$ Z requir. 27 18 the Angle
of Position.

Plate 7. PROBLEM VII.

Fig. 1.

Given the Latitude, Altitude and Azimuth,
to find the Hour of the Day.

Latitude North, —	54 40	} to fi. the H. of the D. by C. the Sev.
Altitude of the Sun, —	47 48	
Azimuth, —	131 52	

I shall project this upon the Plain of the
Horizon, whose Pole or Center is the Zenith,
the

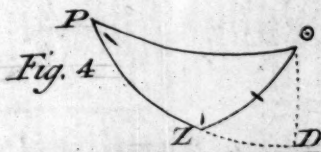
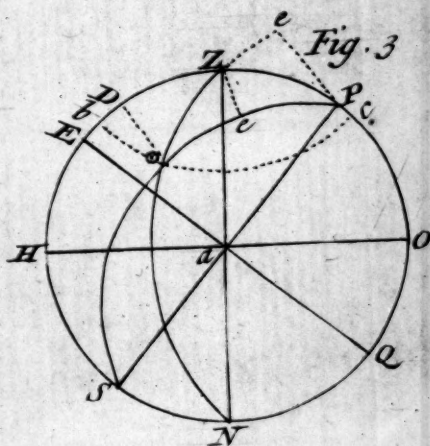
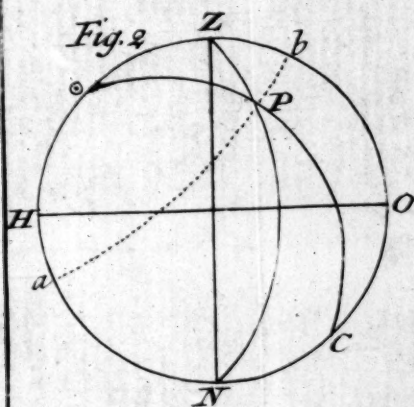
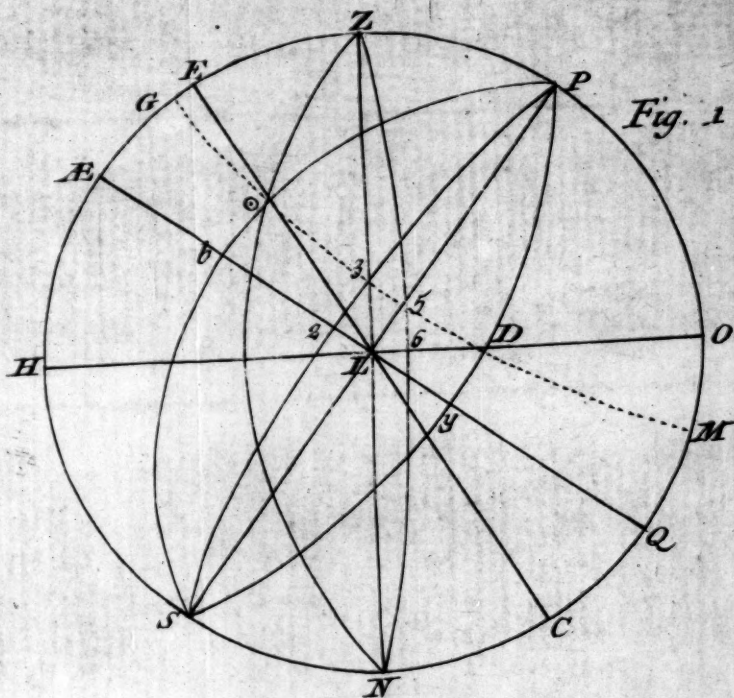


Plate 6.

the Point at which the given Angle falls;
therefore,

$P C Q$, then is $n s$ the Meridian of the Place. Draw the Azimuth $a z b$, to make with the Meridian, the given Angle, $131\ 52$, at z , by Prob. 3, Case 1, of Spherical Geometry; then by Prob. 9, let off the Complement of Latitude, $35\ 20$, from Z to P , and the Complement of Altitude from Z to $\odot$, and through the Points P and $\odot$, draw the Meridian $d P \odot c$, by Prob 5; and then you have finished the Triangle $Z P \odot$, in which there is given $P Z$, $35\ 20$, and $Z \odot$, $42\ 12$, and the Angle $P Z \odot$, $131\ 52$, to find the Angle $\odot P Z$, by Case the Seventh.

Here by Solution the Ninth, the Perpendicular falls without, either from $\odot$ or P : I shall instance in the Former, and then in the Right-angled Triangle, $Z e \odot$, there is given the Angle $\odot Z e$ (the Complement of $\odot Z P$, to 180) $48\ 8$, and the Hypothenufe $\odot Z$, $42\ 12$, to find $Z \odot$, by Case the First.

As Tang. Comp. $\odot Z$, ————	42 12	10.04251
To Radius; ————		10.00000
So Sine Comp. $\odot Z e$ ————	48 8	9.82438

To Tang. $e Z$. ————	31 11	9.78187
-----------------------	-------	---------

G 5

Then

Then by Conclusion the Seventh.

As Sine of the whole $\angle$	66 31	Co ar	0.03754
B se $\angle$ P,			
To Tang. of the Angle $\angle$	48 8	— —	10.04759
$\angle$ Z $\odot$;			
So Sine of the Segment $\angle$	31 11	— —	9.71414
$\angle$ Z,			
To Tang. of the Angle $\angle$	32 12	— —	9.79927
$\odot$ P.			

This 32 12, reduced to Time, gives 2 Hours 9 Minutes, the Hour from Noon required.

plate 7. P R O B L E M VIII.

Fig. 1.

Given the Latitude, Altitude, and Azimuth, to find the Declination.

Latitude North, 54 40	} to find the Sun's Decl. by Case the Eighth.
Altitude, ——— 47 48	
Azimuth, ——— 131 52	

Having projected it, and let fall the Perpendicular, and found the Base Z e, as in Prob. 7, which is there found to be 31 11, proceed by Conclusion the Sixth, thus;

As Sine Comp. Z e, ——— 31 11	Co ar	0.06777
To Sine Com. Z $\odot$; ——— 42 12	— —	9.86970
So Sine Comp. P e, ——— 66 31	— —	9.60041
To Sine Comp. $\odot$ P. ——— 69 49	— —	9.53788
The		

The Complement of Declination, $69\ 49$, subtracted from 90 , gives $20\ 11$, the Declination required.

PROBLEM IX. plate 7.
Fig. 3.

Given the Latitude, Azimuth, and Hour, to find the Declination.

Latitude North, $54\ 40$ }
Azimuth, — $131\ 52$ } to find the Sun's Dec.
Hour. — $32\ 12$ } by Case the Ninth.

To project this upon the Plain of the Meridian, because the given Angles are at the Meridian, and the given Side is an Arch thereof.

P C Q, set the Complement of Latitude, $35\ 20$, from *Z* to *P*, and draw *P S*, and at Right-angles, there to draw *E Q*; then by Prob. 3, Case 2, of Spherical Geometry, draw *P ⊙ S*, to make the given Angle, $32\ 12$, with the Primitive at *P*, and (by the same) draw *Z ⊙ "*, to make the given Angle, $131\ 52$, with the Primitive at *Z*; and where they intersect each other, as at *⊙*, determines the Triangle, *z P ⊙*, in which is given, *z P*, $35\ 20$, and *z P ⊙*, $32\ 12$, and *⊙ Z P*, $131\ 52$, to find *⊙ P*, the Complement of Declination.

Here the Perpendicular (by Solution 9.) must fall upon the Side $\odot Z$, continued, which for Illustration I have annex'd to the Scheme; then in the Triangle $Z e P$, is given $e Z P$, 48 8, and $Z P$, 35 20, to find $Z P e$, by Case the Third, of Right-angled Spherical Triangles.

As Tang. Comp. $P Z e$, ——— 48 8 9.95240
 To Radius; ——— ——— 10.00000
 So Sine Comp. $Z P$, ——— 35 20 9.91158
 To Tang. Comp. $Z P e$. ——— 47 41 9.95918

Then by Conclusion the Eighth.

As Sine Comp. $e P \odot$, — 79 53 Co ar 0.75534
 To Tang. $P Z$; ——— 35 20 ——— 9.85059
 So Sine Comp. $e P Z$, — 47 41 ——— 9.82816
 To Tang. $P \odot$. ——— 69 48 ——— 10.43409

The Complement of Declination being 69 48, the Declination required is 20 12.

Plate 7. PROBLEM X.

Fig. 3.

Given the Latitude, Azimuth, and Hour, to find the Angle of Position.

Latitude

Latitude North, — 54 40 }
 Azimuth, ——— 131 52 } to find the Ang. of
 Hour. ——— 32 12 } Posit. by Case 10.

Proceed in Projection, and Calculation, as in Prob. 9, till you find the Angle $Z P e$, which is 47 41.

Then by Conclusion the Fifth.

As Sine of $Z P e$, ——— 47 41 Co ar 0.13109
 To Sine Comp. $P Z e$; — 48 8 ——— 9.82438
 So Sine Comp. $e P \odot$, — 79 53 ——— 2.99319
 To Sine Comp. $P \odot e$. — 27 19 ——— 9.94866

This 27 19, is the Angle of Position required.

PROBLEM XI. *Plate 7.*
Fig. 2.

Given the Sun's Altitude, Declination, and the Latitude of the Place, to find the Hour of the Day.

Latitude North, 54 40 }
 Declina. North, 20 12 } to find the Hour of the
 Altitude. ——— 47 48 } Day by Case the Elev.

Projection upon the Plain of the Meridian,

PCQ , set the Complement of Latitude, $35^{\circ} 20'$, from Z to P , and draw PS , and at Right-angles thereto, draw EQ , to represent the Equator; draw the Parallel of Declination de , $20^{\circ} 12'$, from the Equator, and the Parallel of Altitude be , $47^{\circ} 48'$, above the Horizon (by Prob. 7, Var. 2, of Spherical Geometry) and where those Parallels intersect each other, (as in $\odot$) is the Point the Sun is in at that Time; then through $\odot$, draw the Meridian $P\odot S$, and the Azimuth $Z\odot n$, and the Triangle $ZP\odot$ is finished, in which you have given, ZP , $35^{\circ} 20'$, $Z\odot$, $42^{\circ} 11'$, and $P\odot$, $69^{\circ} 48'$, to find $ZP\odot$, by Case the Eleventh, the Operation grounded upon Conclusion the Twelfth.

As Tang. $\frac{1}{2}$ the Base $P\odot$, ——— 34.54 0.15638
 To Tang. $\frac{1}{2}$ Sum of the Sides } 38.46 9.90475
 ZP , and $Z\odot$;
 So Tan. $\frac{1}{2}$ Diff. of the same Sid. 3.26 8.77811

To Tan. $\frac{1}{2}$ Diff. Segm. of the Bas. 3.57 8.83924

To the half Base $P\odot$, ——— 34.54
 Add $\frac{1}{2}$ Diff. Segments; ——— 3.57

Sum is the greater Base $\odot e$, ——— 38.51
 Difference is the lesser Base eP . ——— 30.57

Then in the Right-angled Triangle PeZ , is given Pe , 30.57 , PZ , 35.20 , to find the Angle P , by Case the Fourth.

As

As Radius, ————— 10.00000
 To Tang. P ϵ ; ————— 30 57 9.77791
 So Tang. Comp. P Z, ————— 35 20 10.14940

To Sine Comp. Z P Z. ————— 32 14 9.92731

The Angle required, 32 14, converted into Times, gives two Hours 9 Minutes, the Hour from Noon required.

But if the greatest Angle (*viz.* the Azimuth) had been required; then by Axiom the Third, and the Rule deduced there-from.

Z P, ————— 35 20

Z $\odot$, ————— 42.12

P $\odot$, ————— 69.48

Sum, ————— 147.20

$\frac{1}{2}$ Sum, ————— 73.40

————— 69 48

Diff. ————— 3:52

Z P, 35 20, Sine Co ar ————— 0.23782

Z $\odot$, 42 12, Sine Co ar ————— 0.17281

$\frac{1}{2}$ Sum, 73 40, Sine ————— 9.98211

Diff. 3 52, Sine ————— 8.82888

Sum — 19 22 162

Sine Comp. of 65 55 — — — — 9.61081

65 55

Doubled is, — 131 10 the Azimuth required.

Place 7. PROBLEM XII.

Fig. 4.

Given the Azimuth, Hour, and Angle of Position, to find the Latitude.

Azimuth, ——— 131 52
 Hour, ——— — 32 12
 Angle of Position. 27 19 } to find the Latitude
 by Case the Eleven.

In the Projection of this, I shall not trouble the Learner with the common Way of turning the Angles into Sides; but lay it down in its proper Appearance, from what is given, supposing that to be much more instructive to the Learner, and to give him a better Idea of what he is going about; and shall project it upon the Plain of the Horizon.

Draw the Primitive Circle, and cross it with the Diameter ns , to represent the Meridian; then is z the Zenith: Draw the Azimuth eg , to make the given Angle 131.52, at z , by Prob. 3, Case 1, of Spherical Geometry; now because the Angle at P is 32 12, and the Angle at $\odot$ is 27 19; also because the Quantity of Angle is determined by the Distance of the Poles of those Circles, that contain the Angles (by Prob. 9, of Spherical Geometry) it is evident, that the Pole of the Circle to be drawn, must be distant from the Pole of the Circle ns , equal

equal to the Angle at P , viz. $32\ 12$, and from the Pole of the Circle $e\ z\ g$, equal to the Angle at $\odot$, viz. $27\ 19$; therefore draw the Parallel $h\ i$, distant from w , the Pole of $n\ P$, $32\ 12$, equal to the Angle at P (by Prob 7, Case 2;) and (by the same) draw the Parallel $k\ m$, distant from n , $27\ 19$, the Angle at $\odot$, and where these Parallels cut each other, as in o , is the Pole of the Circle to be drawn; then through o , and the Center z , draw the Line or Circle $a\ o\ z\ b$, upon which (by Prob. 9, of Spherical Geometry) set off 90 Degrees from o to r (because every Circle is 90 Degrees distant from its Pole, and through r , draw the Oblique Circle $c\ r\ d$, to cut the Primitive in the opposite Points c and d , it will cut $n\ z\ s$ in p , and $e\ \odot\ g$ in $\odot$, and then is the Triangle $z\ P\ \odot$ finished, and the Side $z\ P$ measured (by Prob. 8, Case 2, is $35\ 20$, the Complement of Latitude required.

But because it is more common for Learners, to project upon the Plain of the Meridian; I shall also give that Variety Plate 7. from the Rules and Reasons before- Fig. 5. mentioned.

$P\ C\ Q$, draw $z\ \odot\ n$, to make the given Angle $131\ 52$, at z , with the Primitive, and find its Pole e , at $27\ 19$, from which, equal to the Angle at $\odot$, draw the Parallel $c\ m\ n$, by Prob. 7, Case 3; and at $32\ 12$, from the Pole of the Primitive (a) draw the Parallel $i\ c\ n$; and where the Parallels cut each other, as in u (or c).

6, if the Triangle was to be projected in the lower Part of the Scheme) is the Pole of the Circle to be drawn; set 90 Degrees from γ to z , and draw the Circle $P r d$, to cut the Primitive in the opposite Points and d , and 'tis done.

The Calculation is the same in both Projections, viz. Take the Complement of the greatest Angle to 180, add that and the two lesser Angles together, and from the half Sum, subtract the Angle opposite to the Side required, and proceed as in Prob. 11.

E X A M P L E.

The greatest Angle,	— — — — —	131 52
Its Complement,	— — — — —	48 8
Angle $z P \odot$,	— — — — —	32 12
Angle $z \odot P$,	— — — — —	27 19
Sum,	— — — — —	107 89
Half Sum,	— — — — —	53 49½
Angle opposite to Side required,	— — — — —	27 19
Remainder,	— — — — —	26 30½

Because it is more common for Learners to draw $z \odot P$ at z , with the Primitive, and find its Pole, at z , from which, equal to the Angle at $\odot$, draw the Parallel $z P$, by Prob. 5; and at z , from the Pole of the Primitive (a) draw the Parallel $z P$; and where the Parallels cut each other, as in a (or

☉ Z P, Co ar	— — — — —	0.12801
Z P ☉, Co ar	— — — — —	0.27337
Half Sum, $53\ 49\ \frac{1}{2}$	— — — — —	9.90699
Remainder,	— — — — —	9.64965
		<u>19.95802</u>

Sine Comp. 17.40 — — — — — 9.97901
 Doubled, is $35\ 20$, the Latitude, or Side Z P required.

Thus much for the Application of Oblique Spherical Triangles in Questions of Astronomy: I shall instance in one Question more, viz. In a Quadrantal Triangle; and then suppose there will be Variety sufficient (with due Application) for the Learner's Improvement, in this Part of Practical Astronomy.

P R O B L E M XIII.

The Sun's Declination, and two different Latitudes under the same Meridian, given to find what Hour of the Day the Sun's Altitude is equal in both Latitudes, and what is the Sun's Altitude that Time.

The two Lat. North, $\left\{ \begin{array}{l} 50\ 0 \\ 70\ 0 \end{array} \right\}$ to find as above.
 Declination North — $23\ 30$

P R O

PROJECTION.

PCQ , with the Lines PS , and EQ ; set the given Latitudes from P to 50, and from P to 70, and draw $H 50$, for the Horizon, to Latitude 50, and $H 70$, for the Horizon, to Latitude 70 (because the two Places have different Horizons) draw rom , equally distant from each Horizon, and draw the Parallel of Declination $t \odot u$, distant from $P 69 48$, equal to the Complement of Declination: Now it is evident, The Sun is some where in the Line rom , because it is equally distant from both the Horizons, and it is in the Parallel $t u$, because that is the Parallel of the given Declination; therefore it must be at the Intersection at the Point $\odot$: Then from 50, set 90 Degrees of the Chords both Ways, to z and N , you have the Zenith and Nadir to the Horizon, $H 50$; set 90 of the Chords also both Ways, from 70 to c and h , which will find the Zenith and Nadir for Latitude 70 00; draw the two great Circles $z \odot N$, and $c \odot h$, they are Azimuths, to each Horizon, the Arches $C \odot$, and $Z \odot$ (being equal) is the Complement of the equal Altitudes in both Latitudes, and the Angle $P o$ the Hour when the Sun's Altitude is equal in both the above-mentioned Latitudes.

Now in order to Calculation, there is given in the Quadrantal Triangle $\odot P O$ (besides the Quadrantal Side PO) the Side $P \odot$, the Complement of Declination, 66 30, and the Angle

☉ 0 P, 30 0 (being the Complement of the Middle between the two Latitudes (which here is 60) to a Right-angle) to find the Angle ☉ P 0, the Hour after 6 in the Morning, or before 6 at Night.

As Radius,	_____	_____	10.00000
To Tang. of ☉ 0 P;	_____	30 0	9.76144
So Tang. Comp. ☉ P,	_____	66 30	9.63830
To Sine ☉ P 0.	_____	14 32	9.39974

Which converted into Time, gives 58 Minutes, viz. 58 Minutes past 6 in the Morning, or 2 Minutes past 5 in the Afternoon.

Then in the Triangle $c P \odot$, there is given $c P \odot$ (the Complement of ☉ P 0) 75 28, and the Side $c P$ 20 0, and $P \odot$ 66 30, to find $c \odot$, by Case the Eighth, of Oblique Spherical Triangles, the Perpendicular (by Solution the Ninth) falling within, viz. From c , upon the Side ☉ P, to the Point n .

As Tang. Comp. $c P$,	_____	20 0	10.43893
To Radius;	_____	_____	10.00000
So Sine Comp. $c P n$,	_____	75.28	9.39957
To Tang. $n P$.	_____	5 13	9.96064

Then

Then by Conclusion the Sixth.

As Sine Comp. $^{\circ} P$, ————— 5 13 0.00189

To Sine Comp. $Z P$; ————— 20. 0 9.97298

So Sine Comp. $\odot$ $^{\circ}$, ————— 61 17 9.68167

To Sine Comp. $Z \odot$, ————— 63 2 9.65645

The Complement of which, 26 58, is the Altitude required, and the Hour before found (2 Minutes past 5.) when the Altitude and Hour is the same in the two given Latitudes.

I need not repeat the Operation for the Triangle $Z P \odot$, because it is evident by Construction, That $Z \odot$ is equal to $c \odot$; nevertheless, for the more certain Proof of the Truth hereof, I shall invert the Question, and take the Altitude (now found) 26 58, the Declination 23 30, and the two given Latitudes 50 and 70, severally, to find the Hour by Case 12, of oblique Spherical Triangles; and first, For Latitude 50.

40 4 0.19193

66 30 0.03760

63 2 9.99818

169 32 9.56853

84 46 19.79624 Sum.

63 2 9.89812 Half Sum: Sine Comp. 37 44

37 44

21 44

Doubled, 75 28

Whose

Whole Complement 14 32 is the Hour
from 6 required.

Then for Latitude 70.

20 0 0.46594

66 30 9.93760

63 12 9.98446

149 32 9.30826

74 46 19.79626 Sum.

63 2 9.89813 Half Sum. Sine Comp. 37 44

37 44

11 44

Doubled, 75.28

Whole Complement, 14 32, is the Hour
from 6, agreeing with the Hour found, by the
fore going Operations.

NOTE, This Problem may be solved by
finding the Time of the Day when the Sun is
in the Prime Vertical, or East or West Azi-
muth, of Latitude 60 (the middle Latitude be-
tween 50 and 70) for then is the Altitude e-
qual in Places under the same Meridian, and
equally remote; therefore you have, as in
Prob. 11. of Astronomy Right-angled, the La-
titude of the Place, and the Sun's Declination
given, to find what Hour and Minute the Sun
will be due East or West by the following
Proportion.

As

As Radius, ————— 10.00000
 To Tang. Comp. Lat. ————— 60 00 9.76144
 So Tang. Declination, ————— 23 30 9.63830
 To Sine of the Hour. ————— 14 32 9.39974

This 14 32, converted into Time, gives 38 Minutes past 6 in the Morning, or 2 Minutes past 5 in the Afternoon, agreeing exactly with the Operation in Page 141; only I thought fit to insert the former Method, because its Solution depends upon a Quadrantal Triangle.



CHAP.

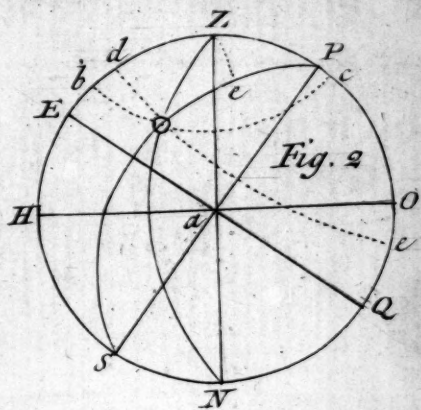
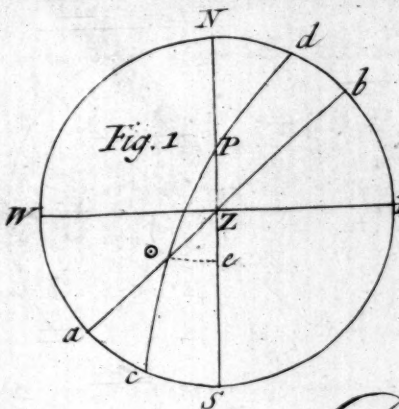
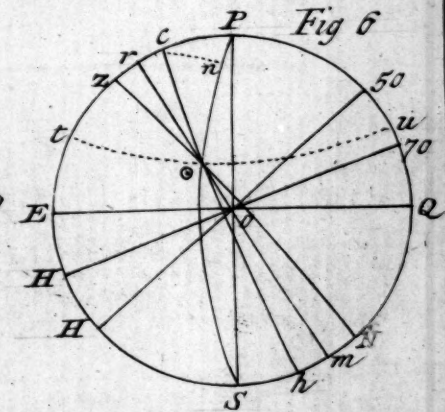
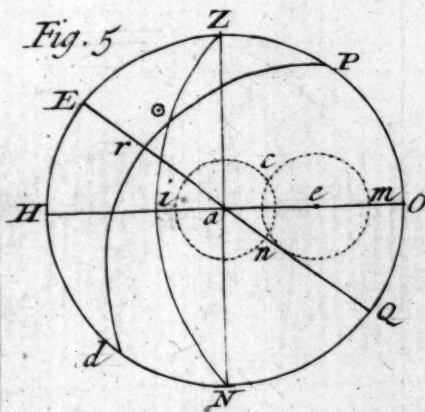
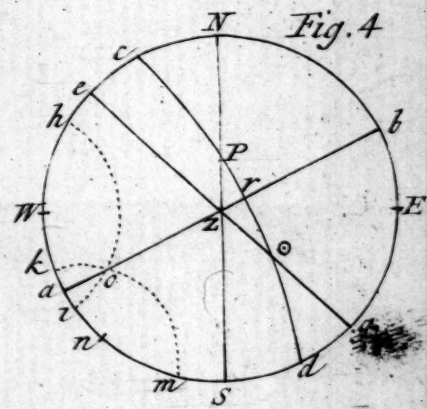
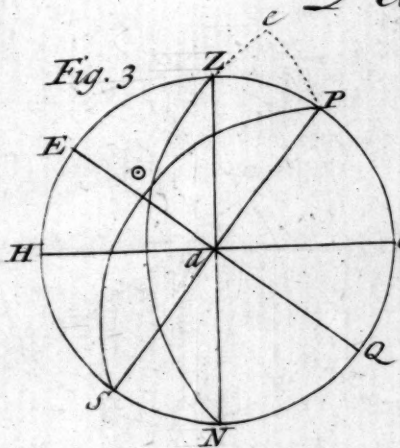
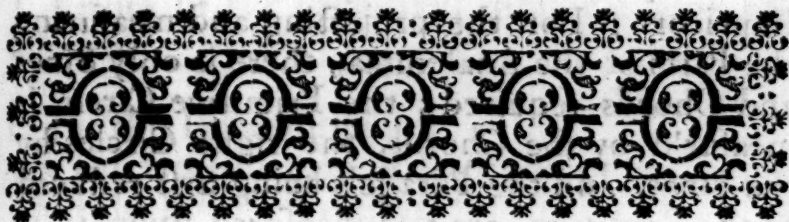


Plate 7





CHAP. V.

Of the ANALEMMA.



SECT. I.

Of the Orthographick Projection of the Sphere, by some call'd the ANALEMMA.



ORTHOGRAPHICK Projection of the Sphere, supposeth the Eye perpendicular to the Plain of that great Circle, upon which it is projected, and at an infinite Distance from it, and is most commonly, and indeed, most intelligibly, perform'd upon the Plain of the Solstitial Colure, viz. the Meridian that intersects

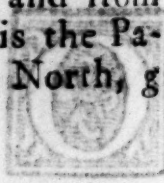
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the

the Ecliptick, in the Tropicks of Cancer and Capricorn: For the projecting and measuring the Parts of the Analemma, there is only Sines and Chords required; the Primitive Circle is measur'd by the Chords, the right Circles by the Sines, and the Parallels being first reduced to the Primitive, are also measured upon it by the Chords.

I have instanced in an Orthographick Projection of the Sphere for Latitude 54 40, North, Sun's present Declination 20 12, North his greatest Declination suppos'd to be 23 30.

Extend your Compasses to 60 of the Chords, and with one Foot in the Center *a*, describe the Circle *Z H N O*, and *Plate 9.* draw *Z N*, the prime Vertical, and *Fig. 1.* *H O*, the Horizon; then set 54 40 (the given Latitude) from *O* to *P*, and from *Z* to *Æ*, and from *H* to *S*, and from *N* to *Q*; and draw *P S* the Axis of the World, and *Æ Q* the Equator; then because the Sun's greatest Declination is 23 30, take 23 30 off the Chords, and set from *Æ* to *E*, and from *Q* to *C*, and draw *E C* to represent the Ecliptick: Set off the Chord of 20 12 the Sun's present Declination, from *Æ* to *b*, and from *Q* to *c*, and draw the Line *b c*, which is the Parallel of Declination for 20 d. 12 m. North, *g h* is a Parallel of Altitude.



Arch's

Arches of the Primitive, measur'd upon the Chords.

- $O P$ is the Latitude 54 40,
 $P Z$ the Complement of Latitude 35 20,
 $\mathcal{A} E$ the Sun's greatest Declination 23 30,
 $\mathcal{A} b$ the present Declination 20 12,
 $K O$ the Sun's Altitude at 6 a-clock 16 21.

Arches of right Circles measur'd on the Sines.

- $a e$ the Sun's present Declination 20 12,
 $a f$ the Sun's Amplitude 38 40, from the East,
 $a \odot$ the Sun's Longitude 60 00,
 $a d$ the Sun's Altitude when East or West 25 3,
 $a i$ the Sun's Altitude when in the Point $\odot$ 47 48,
 $a n$ (being equal to $e g$) is 16 21 the Sun's Altitude at 6.

Arches of lesser Circles to be reduced to the Primitive, and measur'd by the Chords.

- $g \odot$ the Sun's Azimuth from the South,
 $\odot i$ the Sun's Azimuth from the East, } in the Point $\odot$.
 $d e$ the Hour from 6 that the Sun is due East 15 7,
 $n e$ the Sun's Azimuth from the East or West at 6 12 1,
 $e f$ the Ascensional Difference 31 16,
 $\odot e$ the right Ascension 57 48.

Before I proceed to any Astronomical Problems, it will first be necessary that the Learner understand how to reduce a lesser, or Parallel Circle to the Primitive, so as any Part of the Parallel may be measur'd upon the Chords.

1. Take half the Parallel in your Compasses, and set off from the Center of the Primitive upon any right Circle, where there is most Room, or Conveniency; and where the other Foot of the Compasses fall, make a Mark.

2. Take in the Compasses, the Part of the Parallel to be measur'd; and with one Foot in the aforesaid Mark, turn the other Foot about, and make an occult Arch.

3. Lay a Ruler from the Center of the Primitive, so as just to touch the Extremity of the said Arch, and observe where the Ruler cuts the Primitive, making a Mark.

4. Extend your Compasses from the Mark upon the Primitive, to the Point where the right Circle, by which the Work was done, cuts the Primitive; that Extent measur'd on the Chords, gives the Quantity of the Arch of the Parallel requir'd.

E X A M-

EXAMPLE.

I would reduce the Parallel $k l$, to *Plate 9.*
the Primitive, in order to measure *Fig. 2.*
the Arch $i l$, of the said Parallel.

Take the Extent $l k$ in your Compasses, and set it from a to c , upon the right Circle $d a m$, then take $i l$ in your Compasses, and with one Foot in c , describe the Arch g ; a Ruler laid from a , to touch the Extremity of the Arch g , will cut the Primitive in e , and the Arch $d e$ measur'd on the Chords, gives the Quantity of the Arch $i l$ requir'd.

NOTE, To set off any Number of Degrees upon a Parallel, or lesser Circle, is only the Reverse of this:

EXAMPLE.

It is required to set 40 Degrees from r upon the Parallel $r t$:

Take 40 of the Chords, and set from m to s , and draw the Line $a s$; set $r t$ from a to x , then with one Foot of your Compasses in x , take the nearest Distance to the Line $a s$, and with that Extent, and one Foot in r , the other will reach to u , and $r u$ is 40 Degrees in that Parallel.

S E C T. II.

Problems in Astronomy solved by the Orthographick Projection.

Plate 9.

P R O B L E M I.

Fig. 1.

Given { The Sun's Long. 60 0 } Requ. Sun's
 { Greatest Declin. 23 30 } present Dec.

I shall in this, as in the same Problem of Stereographick, project it for the Latitude 54 40, that the same Scheme may serve in other Cases, where the Latitude is given or requir'd.

Having drawn the Axis of the World, the Prime Vertical, the Equator, Ecliptick and Horizon, as before directed; set the Sine of 60 00, the given Longitude from *a* to $\odot$, and from $\odot$ take the nearest Distance to the Equinoctial *Æ Q*, at which Distance draw the Parallel of Declination *b c*, it cuts the Primitive in *b* and *c*, the Arch *Æ b*, or *Q c* measur'd on the Chords, or *a c* measur'd on the Sines, gives 20 12 the Declination requir'd.

Plate 9.

P R O B L E M II.

Fig. 1.

Given { Sun's Longitude 60 0 } Requ. right
 { Greatest Declin. 23 30 } Ascension.

In the Scheme as projected before the Arch $\odot$ of the Parallel of Declination *b c*, reduced

to the Primitive as before directed, gives 57 48 the right Ascension requir'd.

NOTE, The right Ascension is properly an Arch of the Equator, intercepted between the nearest Equinoctial Point, and the Point where a Meridian drawn through $\odot$, cuts the Equator; but in this Projection, the Meridians between the Primitive and its Center, not being Circles, or Parts of Circles, are not to be attain'd but by Approximation, viz. by finding several Points through which they are to be drawn, which takes up a great deal of Time, and renders the Work very troublesome; and this Way of Reckoning the right Ascension upon the Parallel being exact, and much more easy, I have rather made Choice of it the same Way, being equally useful in other Cases.

PROBLEM III.

Plate 9.

Fig. 1.

Given $\{ \odot$'s greatest Dec. 23 30 } to find Lon.
 $\{ \odot$'s present Dec. 20 12 } right Ascen.

Draw the Primitive, the Axis, Horizon, Equator, Prime Vertical, &c. as before taught, and set 23 30 the greatest Declination from $\mathcal{A}$ to E , and from Q to C , and draw EC the Ecliptick; set the present Declination 20 12 from $\mathcal{A}$ to b , and from Q to c , and draw the Parallel of Declination bc , and where that cuts the Ecliptick, as in $\odot$, is the Point the Sun is

in, and the Arch of the Ecliptick $a \odot$, measured on the Sines, is 60 Degrees the Longitude of the Sun required, and the Arch $e \odot$ 57 48, is the right Ascension.

Plate 9. PROBLEM IV.

Fig. 1.

From the Scheme projected according to the Directions given in the Beginning hereof, for Latitude 54 40, and Sun's Declination 20 12, to find,

1. The ascensional Difference.
2. The Sun's Amplitude.
3. What Hour and Minute the Sun will be due East or West.
4. Sun's Altitude at 6.
5. Sun's Altitude when East or West.
6. Sun's Azimuth at 6.

1. The Arch $\odot e$ of the Parallel bc , measured the fore-going Rules, is 57 48 the ascensional Difference; the Use of which, for finding the Time of Sun's rising and setting, and the Length of the Day or Night. See Prob. 8. and 9. of Astronomy.

2. The Arch $a f$ of the Horizon measur'd on the Sines, is 36 40 the Sun's Amplitude from the East or West.

A H

3. The

3. The Arch de of the Parallel bc , is 15 7, which reduced to Time, gives one Hour after 6 in the Morning, or one Hour before 6 in the Evening, viz. 7 in the Morning, or 5 in the Afternoon.

4. For the Sun's Altitude at 6, it should be represented by eq , which is the Arch of an Azimuth; but there being great Difficulty, both in projecting and measuring the Azimuths in this Projection, and the Arch na of the prime Vertical being equal to eq , (because between the same Parallels, and cutting them at right Angles,) na measur'd on the Sines, is 16 21 the Sun's Altitude at 6 required.

5. The Sun's Altitude when East or West, is represented by ad , which measur'd on the Sines, is 25 Degrees 3 Minutes.

6. The Arch ne of the Parallel of Altitude kl , is 12 Degrees 1 Minute the Sun's Azimuth at 6, reckoned from the East or West Northerly, and its Complement ek , is the Azimuth from the North.

PROBLEM V.

Plate 9.

Fig. 1.

If at Sea you cannot have an Observation for the Latitude at Noon, but by the Benefit of a clear Evening, having the Declination given

H 5

you,

you can find the Amplitude; and by the Help of that, and the given Declination, you desire to know your Latitude.

Suppose the Amplitude at Sun's setting 36 40 from the West Northerly, and the Declination 20 12 North to find the Latitude.

With the Chord of 60, draw the primitive Circle, and cross it at right Angles, with ZN the prime Vertical, and HO the Horizon; set the Sine of 36 40 the given Amplitude from a to f , then take in your Compasses the Sine of the Declination 20 12, and with one Foot in f , sweep the small Arch r ; then through the Center a , and by the Extremity of the Arch r , draw $\mathcal{E} a Q$, which represents the Equator; and at right Angles thereto, draw PS ; then the Arch PO measur'd on the Chords is 54 40 the Latitude sought.

Plate 9.

PROBLEM VI.

Fig. 1.

Or if you cannot get the Sun's Altitude at Noon, but have an Opportunity to obtain the Sun's Altitude at 6, either Morning or Evening, which suppose to be 16 21, and the Declination 20 12 North, to find the Latitude.

Having drawn the primitive Circle, and cross'd it with the Diameters, ZN the prime Vertical, and HO the Horizon; set the Chord
of

of 16 21 the given Altitude from H to l , and from O to k , and draw the Parallel of Altitude lk ; then take in your Compasses the Sine of the given Declination 20 12, and with one Foot in a , with the other cross the Parallel lk , in e , through z , and the Center a , draw the Line $P a S$; the Arch $P O$ measur'd on the Chords is 54 40 the Latitude required.

PROBLEM VII.

Plate 9.

Fig. 1.

Given the Sun's Declination 20 12 North, and his Altitude when East or West 25 3, to find the Latitude.

Draw the Primitive, and cross it as before; set the Sine of the Sun's Altitude when East or West, viz. the Sine of 25 3 from a to d ; then with the Sine of 20 12 the Declination, and one Foot in a , sweep the small Arch e ; then by the Extremity of that Arch, and through the Point d , draw the Parallel of Declination be , and parallel thereto through the Point a , draw the Equinoctial $AE a Q$, and at right Angles to it draw the Axis $P a S$; the Arch $P O$ measur'd on the Chords gives 54 40 the Latitude.

Plate 9. PROBLEM VIII.

Fig. 1.

You may also from this Projection solve those Problems that depend upon Oblique Spherical Triangles; as suppose there is

Given $\left\{ \begin{array}{l} \text{Sun's Altitude, } 47^{\circ} 48' \\ \text{Declin. North, } 20^{\circ} 12' \\ \text{Azi. from East, } 41^{\circ} 52' \end{array} \right\}$ to find the Latit. and Hour.

Draw the Primitive, and cross it with the prime Vertical and Horizon, as before, and set the Chord of the Altitude $47^{\circ} 48'$ from H to g , and from O to h , and draw $g h$ for the Parallel of Altitude; then by the Directions given for measuring and setting of Degrees upon a Parallel, set $41^{\circ} 52'$ the given Azimuth from i to $\odot$, and with the Sine of the given Declination, and one Foot in $\odot$, make the small Arch x , through the Extremity of which, and the Center a , draw $A a Q$ the Equinoctial, (because the Sun's Declination is his nearest Distance from the Equinoctial,) and parallel thereto, through the Point $\odot$, draw the Parallel of Declination $b e c$, also at right Angles with the Equinoctial, draw the Axis $P a S$; the Arch $P O$ is the Latitude sought, and the Arch $\odot e$ of the Parallel $b e c$ is $57^{\circ} 48'$ the Hour from 6, which allowing 15 Degrees to an Hour, and one Degree to 4 Minutes, gives 3 Hours 51 Minutes after 6 in the Morning, or before 6 in the

the Evening, viz. 51 Minutes past 9 in the Morning, or nine Minutes past two in the Afternoon.

PROBLEM IX.

Plate 9.

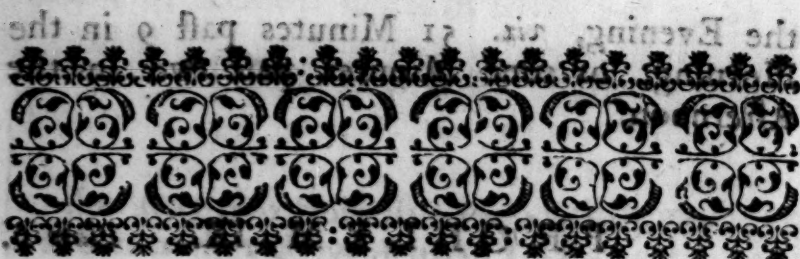
Fig. 1.

Given $\left\{ \begin{array}{l} \odot\text{'s Altitude, } \text{---} 47^{\circ} 48' \\ \odot\text{'s Declin. North, } 20^{\circ} 12' \\ \text{Latitude North. } \text{---} 54^{\circ} 40' \end{array} \right\} \begin{array}{l} \text{to find the} \\ \text{Azimuth} \\ \text{and Hour.} \end{array}$

Draw the Primitive, and cross it with the Horizon HO , and the prime Vertical ZN ; set the Latitude $54^{\circ} 40'$ from O to P ; and draw PS , and at right Angles thereto draw the Equinoctial AEQ ; set the Chord of Declination $20^{\circ} 12'$ from Q to c , and from AE to b , and draw the Parallel of Declination bc ; likewise set the Altitude $47^{\circ} 48'$ from H to g , and from O to h , and draw the Parallel of Altitude gh ; and where that cuts the Parallel of Declination, as in the Point $\odot$, is the Place the Sun is in; and $\odot i 41^{\circ} 52'$, is the Sun's Azimuth from the East or West; and $\odot e 57^{\circ} 48'$ is the Hour, as in Prob. 8.

I might have instanced in more Problems of this kind, but I think what I have already done, is sufficient to instruct the Learner in the Nature and Use of this Projection.

CHAP.



CHAP. VI. Of DIALLING.

SECT. I.

How to project a DIAL Stereographically upon any Plain.

THE Projection of any Plain Dial, is no more but the Projection of the Sphere, upon the Plain of that great Circle, to which the Dial is Parallel, the Meridians at every 15 Degrees Distance representing the Hour Lines, and the Axis of the World (terminating at the two Poles) representing the Stile; as the following Examples will make evident.

SECT.

S E C T. II.

How to project an Horizontal DIAL.

AN Horizontal Dial, is so called, because it is parallel to the Horizon, and therefore is projected upon the Plain thereof, to the Latitude given.

E X A M P L E.

It is required to draw an Horizontal Dial for the Latitude of London, 51 Degrees 32 Minutes.

First, Draw the Primitive Circle *N W S E*, to represent the Horizon, and cross it at Right-angles, with the Lines *N S*, and *W E*, then the Center *Z* is the Zenith of the Place (because it is the Pole of the Horizon) and the Points *N, S, E, W*, are the North, South, East and West Points of the Projection.

Then because the Pole of the World is distant from the Zenith $38^{\circ} 28'$ (equal to the Complement of Latitude) set of $38^{\circ} 28'$ upon the Right Circle *S N*, by Prob. 9. from *Z* to *P*; then is *P* the Pole: Also because the Equator is distant from the Zenith $51^{\circ} 32'$ the Latitude

titude given, set off $51^{\circ} 32'$ from Z to Q , by Prob. 9. of Spherical Geometry; and draw the Oblique Circle $W Q E$, to represent the Equator; then from Q to a , set off 15 Degrees each Way upon the Equator, by Prob. 9. and from Q to b , 30 Degrees each Way, and from Q to c , 45 Degrees, and to d , 60 Degrees, and to e , 75, as in the Figure; then through P , and the several Points $a b e$, &c. draw the Meridians 1 $P a$ 1, and 2 $P b$ 2, &c. by Prob. 5; which done, draw Lines from the Center Z , to the Points 1, 2, 3, &c. these are the Hour Lines required.

Then for the Stile's Height, measure the Arch $P N$, of the Right Circle $S N$, by Prob. 8. the Quantity of that Arch is the Height of the Stile required.

NOTE, If you draw your Projection upon the same Plain (whether Wood, Stone, &c.) that you intend to make your Dial upon, 'tis best to do it obscurely, that when you have found the Hours Distances upon the Periphery, the other Circles, &c. may be done out at Pleasure, and the Stile may be laid down, by seting $51^{\circ} 32'$ of the Chords from N to B ; then draw the Line $Z R$, and $R N$; but you may make the Lines $Z R$, and $Z N$, longer, or shorter, at Pleasure; only the Angle at Z is to be always equal to the Latitude of the Place: Thus a Piece of thin Plate, &c. of the Form of the Triangle $Z R N$, shall be the Stile required.

To

To find the Hours Distances by Trigonometrical Calculation, the Proportion is,

As Radius,

To Sine of Latitude;

So Tangent of 15 Degrees,

To Tangent of the Hours Distance of 11 and 1 from the Meridian :

And so Tangent of 30 Degrees,

To Tang. of the Hours Distance of 10 and 2 ;

And so of 45 Degrees,

For 9 and 3, &c.

This Proportion you have in many Books of Dialling; but no Reason given for it, nor Demonstration of it, as you have from this Projection, in which, in the Triangle $P N 1$, you have given $P N$, the Latitude $51^{\circ} 32'$; and the Angle at P , 15 Degrees, to find the Side $N 1$, the Distance upon the Primitive Circle, between the Hours of 12 and 1 (or 12 and 11, which is the same;) and you may find it by the Globical Projection, taught in the former Part of this Book; for *Plate 8. Fig. 3.* supposing the Triangle $P N 1$, to represent the Triangle $P N 1$ in the Projection, you will find by the Rule laid down in Globical Projection, that the Proportion lies in the Mansion $P b c$, and is,

As

As (P b) Radius, ————— 90.00 10.00000
 To (P n) the Sine of Latitude; 51 32 9.89374
 So (b c) the Tangent of 15 Degrees } 9.42805
 the Angle at P,
 To (N 1) the Distance upon the
 Primitive Circle between the Hor }
 of 12 and 1, viz. 11° 51', whose } 9.32179
 Tangent is, : 1 from the Meridian

By the same Proportion with Tangent of
 30 Degrees, you find the Hour Distances of
 10 and 2, and by 45 Degrees, the Hour of 9
 and 3, &c. which is an intelligible Demon-
 stration of that common given Proportion a-
 bove mentioned.

S E C T. III.

*How to project an Erect Direct
 South DIAL.*

THIS Dial is parallel to (and therefore
 projected upon the Plain of) the Prime
 Vertical, or East and West Azimuth.

EXAMPLE.

'Tis required to draw a South Dial for the Latitude of $55^{\circ} 8'$.

First, Draw the Primitive Circle $W Z E N$, to represent the Prime Vertical; then cross it at Right-angles, with the Line $z N$ (which in this Case is the Meridian of the Place) and $E W$, the Horizon; then is z the Zenith, N the Nadir, E East, and W the West Points of the Projection; then because the Equator is $34^{\circ} 52'$ above the Horizon, equal to the Complement of Latitude, set off $34^{\circ} 52'$, from S to Q , by Prob. 9. and draw the Oblique Circle $W Q E$, to represent the Equator: Also because the South Pole (which in this Case is the visible Pole) is below the Horizon $55^{\circ} 8'$, equal to the Latitude, set off $55^{\circ} 8'$, from S to P , by Prob. 9. then is P the South Pole; this done, set off upon the Equator $15:30:45$ Degrees, &c. both Ways, from Q to $a b c$, &c. and draw the Meridians $1 P a 1$, $2 P b 2$, &c. to cut the Primitive Circle in the Points $1 2 3$, &c. then Lines drawn from the Center S , to the Points $1 2 3$, are the Hour Lines required.

The Height of the Stile is always equal to the Complement of Latitude, which here is $34^{\circ} 52'$, and may be made as directed in the Horizontal Dial, the Angular Point of the Stile both in this and Horizontal, being

ing fixed at the Center of the Dial, and the bottom of the Stile proceeding upon the Meridian Line towards the Hour of 12.

A North Dial is only projected on the contrary Side of the Prime Vertical; but the Hour Distances, Height of the Stile, &c. is all the same; only in a North Dial, the Hour of 12 is put upwards, and the Angular Point of the Stile is put downwards; and therefore to avoid Prolixity, I shall say no more about the Projection of it.

The Proportion for finding the Hour Distances on a South Dial, by Trigonometry, is,

As Radius,

To Sine Complement of Latitude;

So Tangent of 15 Degrees,

To Tangent of the Distances of the Hours of 11 and 1 from the Meridian;

And so by the Tangent of 30 Degrees, 45 Degrees, 60 Degrees, &c. you find the Hour Distances of 10 and 2, of 9 and 3, and of 8 and 4, &c. by the Rules laid down in the Horizontal Dial, by which you may also find the Reason of the Proportion in a South Dial, in the Triangle $P N 1$, by the Directions before-mentioned.

S E C T.

S E C T. IV.

How to make an East or West DIAL:

BEFORE you can project this Dial, you must be determined how high your Stile shall be, which may be done at Pleasure, according to the Largeness of your Plate, or Plain, which is most proper to be an Oblong, or long Square, upon which draw a Line right-up the Middle, as $a b$; then cross it at Right-angles, with the Line $c d$; which done, take in your Compasses the Height of your Stile, and with one Foot in the Crossing of the aforesaid Lines, with the other sweep the Circle $c e d h$, which divide into 24 equal Parts, and draw the two Tangent Lines $k m$, and $n o$, Parallel to the Line $a b$, and just to touch the Circle $c e d h$, in c and d ; then is the Line $c d$ the Hour Line of 6, upon which the Stile must stand; then from the Center q , lay a Scale over each Division of the Circle, and observe where it cuts *Plate 8.* the Lines $k m$, and $n o$; and from *Fig. 4.* these Marks, draw the Lines 5, 5, 7, 7, &c. which shall be the Hour Lines required.

The

The Dial thus finished, is fit for any Latitude; only when you put it up, you must have it so situate, that the Line $c d$ may just point to the Poles, then will $a b$ be Parallel to the Equinoctial; and so place it upright upon the East Side of a Wall, &c. the Height of the Stile equal to the Semidiameter of the Circle by which it is projected; and then is your Dial ready for Use.

A West Dial is the same; only instead of 5, 6, 7, 8, &c. 'tis numbred 8, 7, 6, 5, &c. but always the 6 a Clock Line is that that cuts the Circle in 2 equal Parts, and passes over its Center.

The Trigonometrical Proportion for finding the Hour Distances, is,

As Radius,

To the Height of the Stile;

So Tangent of 15 Degrees,

To the Distance 6 7:

And so Tangent of 30 Degrees,

To Distance 6 8;

And so Tangent 45 Degrees, or Radius,

To Distance 6 9, &c.

Hence the Hour Line of 9, should always just touch the Circle; because Tangent 45 Degrees is equal to Radius; and the Reason of this Proportion is so apparent to any that understand

derstand the Construction of the Tangents, that it is needless to insist any more upon it.

S E C T. V.

How to project a Declining DIAL.

IT is required to project a Dial for Latitude, $51^{\circ} 32'$, declining from the South Westward 30° Degrees.

First, Draw the Primitive Circle $b c d e$, and cross it with two Diameters $b d$ and $c e$; then because the Declination is West 30° Degrees, draw the Oblique Circle $b n d$, to represent the Meridian of the Place, to make an Angle of 30° Degrees, with the right Circle $b a d$ (the Meridian of the Plain (by Prob. 3. Case 2.) and because the Pole of the World is always in the Meridian of the Place $b n e$, and distant from the Horizon (*viz.* below it) equal to the Latitude; draw the Parallel ($b i k$) distant from the Horizon ($c a e$) $51^{\circ} 32'$, and where it cuts the Meridian $b n d$ (as in m) is the Pole of the World; through m draw $a m g$, and now you have the two Right-angled Triangles $m g d$, and $m n a$, and the Quadrantal Triangle $a m d$,

m, d , to find the Requisites in your Dial, and the Reason of the Proportions used for that End, for the several Arches and Angles in these three Triangles, measur'd by the Rules laid down in the Geometrical Part hereof, gives the Quantity of the Requisites by them represented; for the Circle $b n d$ represents the Meridian of the Place, $b a d$ the Meridian of the Plain, $a m g$ the Subtile, $m g$ the Stile's Height, $a i$ the Latitude, *etc.* and the Angle $i d m$, the Declination of the Plain $m d g$, its Complement $d g$ (the Measure of the Angle $i a m$) is the Subtile's Distance from the Meridian $a n$ (the Measure of $a d n$) the Declination, *etc.*

Then first, For the Substile's Distance from the Meridian, represented by the Side $d g$, in the Triangle $m g d$, in which you have given $m g d$ the Complement of Declination, and $m d$ the Complement of Latitude to find $d g$, the Subtile's Distance from the Meridian, which by my Lord *Nepair's* Proportion, is,

As Tangent Latitude, — — —	51:32	10.09991
To Radius; — — — — —	90:00	10.00000
So Sine Declination, — — —	30:00	9.69897
To Tangent Subst. Dist. — — —	21:40	9.59906

Or in the Quadrantal Triangle $a m d$, the Angle $d a m$; or in the Triangle $m n a$, the Complement of the Angle at a is the same; but shall leave the Operations for the Reader's Practice.

Secondly,

Secondly, For the Stile's Height in the Triangle $m g d$, the Side $m g$ is required from the former Data, and is by the Gnomonical Projection,

As Radius,	—————	90:	0:	1.000000
To Sine Complement Dec. S. C.	30:	0.	9.93753	
So Sine Complement Lat. S. C.	51:32		9.79383	
To Sine of the Stile's Height.	32 56		9.73136	

Or in the Quadrantal $a m d$, or the Right-angled Triangle $m n a$, the Side $a m$ common to both, is the Complement of the Stile's Height required.

Thirdly, For the Difference in Time between the Meridian and Substile, which is the Angle $d m g$, it is found by the Globical Projection to be,

As Sine Latitude,	—————	51:32	9.89374
To Radius;	—————	90:00	10.00000
So Tangent of Declination,	—	30:00	9.76144
To Tang. of Diff. in Time, &c.	36 24		9.86770

Which 36 24, converted into Time, is 2 Hours 26 Minutes, between the Meridian of the Plain, and the Substile; and if you draw all the Meridians through the Pole m , and making an Angle with each other of 15 Degrees, the Points where they intersect being mark'd with 12:1:2:3, &c. and Lines drawn from the Center a to each Mark, shall be the Hour Lines required.

I have not drawn all the Meridians, lest it should too much confuse the Diagram; but I have drawn the Hour Line, of 6 (*viz.* 6 *m.* 6.) by which you may do all the rest by Prob. 3. Case 4. Var. 2. of Spherical Trigonometry, making an Angle of 15 Degrees with each other at the Pole *m.*

Thus you have not only the Proportions given, which you meet with in most Books of Dialling; but also the Reasons of them, as deduced from the known Principles of Spherical Trigonometry, and Projection of the Sphere, the true Foundation of which being known, 'tis very easily apply'd to any Case, either in Astronomy, Dialling, or Geography, or any Part of the Mathematicks, that depend upon Spherical Triangles.

S E C T. VI.

How to draw a South DIAL, or a Horizontal DIAL upon their respective Plains without the Help of any Mathematical Calculation.

1. **A** Dial parallel to the Plain of the Equinoctial, is only a Circle divided into 24 equal Parts for the Hours; and the Stile or Axis is erected from the Center perpendicular to the Plain.

2. The

2. The Plain of an Equinoctial Dial, makes an Angle with the Plain of a South Dial, for any given Latitude equal to the said Latitude; and with a Horizontal Dial, the Equinoctial Dial makes an Angle equal to the Complement of the Latitude, for which the Equinoctial Dial is made.

3. All Dials, however situate, have their Stiles parallel to each other (because all parallel to the Axis of the World) consequently if a South Dial was erected upon the North End of a Horizontal, one Stile would serve for both, making an Angle with the Horizontal Dial equal to the Latitude, and with the South Dial equal to the Complement of the Latitude of the Place, the two Plains making a right Angle with each other.

Hence also if a South Dial was erected at the North End of an Equinoctial Dial in North Latitude (or a North Dial at the South End in South Latitude) and contiguous to it, one Stile would likewise serve for both, making right Angles with the Plain of the Equinoctial Dial, and an Angle equal to the Complement of Latitude with the Plain of the South Dial (the two Plains making an Angle with each other equal to the Latitude.)

From these Considerations, a general Rule may be deduc'd, for transferring the Hour Lines from an Equinoctial Dial, to a South Dial, or to a Horizontal Dial, the equal Divisions in one becoming unequal in the other; as for Instance,

Plate 10. It is required to make a South
Fig. 3. Dial for Latitude 40 Degrees North.

First, Draw the Line ec , suppos'd Perpendicular to the Horizon, to represent the Plain upon which the South Dial is to be made; then from c , draw the Line ca , to make Angle of 40 Degrees with the Line ce (because the Plain of the Equinoctial Dial makes an Angle of 40 Degrees with the Plain of the South Dial) then from any convenient Point in the Line ac , erect the Perpendicular bd , to represent the Stile (because the Stile is Perpendicular to the Plain of an Equinoctial Dial) and that will also represent the Stile of the South Dial, making an Angle with the Plain of it at d , equal to the Complement of Latitude, viz. 50 Degrees. This done with the

Extent bc , and one Foot in the Center n , describe the Circle $gh129$, which divide into 24 equal Parts in the Points $12\ 1\ 2\ 3$, &c. Continue

the Line $gn12$ to l , and at right Angles therewith, draw the Line op , so as just to touch the Circle in the Point 12 , and produce the Lines $n4$, $n3$, $n2$, &c. to cut the said Line in ors , &c. then from the Line ce (Fig. 3.) which represents the Plain of the South Dial, take the Extent dc (which represents the Semidiameter of the South Dial) and with one Foot in the Point 12 , where the Line op cuts the Line gl , with the other find the Point q , upon which as a Center with the same Extent $q12$, describe the Circle $12klm$, and from the Points rst , &c. draw the Lines rq , sq , tq , &c. and where

where these cut the Circle $12 k l m$, as in 4 3 2 1, &c. are the unequal Divisions representing the true Hour Lines upon the South Dial requir'd, the Stile fixed at the Point q making an Angle of 50 Degrees with the Plain of the Dial equal to the Complement of the propos'd Latitude.

The Reason of this Method may yet be better conceiv'd, if you imagine this Paper upon which the Projection is drawn, to be folded in the Line $o p$, so that the Plain $g 12$ may make an Angle of 40 Degrees (the given Latitude) with the Plain $12 l$; for then the Perpendicular rais'd from n the Center of the lesser Circle, will fall in q the Center of the greater Circle, as is evident from the Figure $a b c d e$, Fig. 3.

For those that understand Trigonometry, observe in the Triangle $b c d$, (Fig. 3.) the Angle c is the Angle that the two Plains make with each other, equal to the Latitude, and consequently the Angle d is equal to the Complement of Latitude; because the Angle b is a right Angle. The Side $b c$, is the Semidiameter of the Equinoctial Dial, which may be assumed of any Length at Pleasure, and $d c$ is the Semidiameter of the South Dial; for the finding of which, the following Proportion is deduced from the said Triangle.

As Sine of the Angle $b d c$,
To Side $b c$;
So Radius,
To Side $d c$.

Therefore,

As Sine Complement of Latitude,
To Radius;

So the Semidiam. of the Equinoctial Dial,
To the Semidiameter of the South Dial.

And in this, as in all other Cases, having found the Hour Lines, you may put the Center q to the upper Part of the Dial, and reduce it to a Square, or an Oval, &c. and add to it what Ornament you think fit.

NOTE, Always take Care to give Allowance for the Thickness, or Substance of your Stile, by imagining the Line upon which it stands to be split down the Middle, and the Parts removed asunder, as far as the Thickness of the Stile; for if you lay the Meridian, or Subtile only under the Middle of the Stile, your Dial will be too slow when the Sun is on the East Side of the Stile, and too fast when on the West Side, sometimes by a Quantity of the Shadow (nearly) equal to half the Thickness of the Stile.

The Horizontal Dial may by the same Means be projected from the Equinoctial Dial; as suppose I would make a Horizontal Dial, for the Latitude of 40 Degrees, having
Plate 10. drawn the Line Dc , to represent
Fig. 2. the Plain parallel to the Horizon; then from c draw the Line ca , to make with the Line Dc , an Angle of 50 Degrees at c , equal to the Complement of Latitude; then from any Point at b , in the Line ac , draw the Line $b\odot$, perpendicular to ac ; then is $b\odot$ the Stile to both Dials, viz. the Equinoctial

quinoctial Dial $a c$, and the Horizontal Dial $\gg c$; then with the Extent $b c$, draw a Circle, which divide as before, into 24 equal Divisions; and upon the 12 a Clock Line, continu'd with the Extent $\odot c$, and one Foot in 12, find the Center of the greater Circle, and with the same Extent draw the Circle to represent the horizontal Dial, and so proceed in all respects as before, to transfer the equal Divisions in the lesser Circle to the tangent Line, and from thence to the greater Circle's Center, and they will make the unequal Divisions upon the Periphery of the greater Circle, which shall describe the Hour Lines requir'd, which being done in all respects, as in the South Dial, I shall not need to make any further Repetition.

The trigonometrical Proportion for finding the Diameters of the greater Circle, deduc'd from the Triangle $\odot b c$, is;

As Sine of $b \odot c$,
To Radius;
So the Side $b c$,
To the Side $\odot c$.

That is,

As Sine of Latitude,
To Radius;
So the Semidiameter of the lesser Circle,
To the Semidiameter of the greater.

NOTE, You need not project any more than 6 Hours, viz. from 12 to 6; for Hours equally distant from 12, are equally distant from one another, viz. the Distance between

the Hour Lines of 9 and 10, is equal to the Distance between those of 2 and 3, and the same of those equally distant from 6, as from 7 to 8, is the same Distance as from 4 to 5, and the opposite Hours are also equally distant, as from 4 to 5 in the Morning, is the same Distance upon the Circle, as between 4 and 5 in the Afternoon.



C H A P. VII.

Of GEOGRAPHY.



THE principal Subject of this Treatise being Trigonometry, and the Application thereof to Practice, my Intent in this Chapter, is to shew the Use of Spherical Trigonometry, when apply'd to Problems in Geography; in which I shall not have Occasion for many.

DEFINITIONS.

Latitude of a Place, is an Arch of the Meridian, intercepted between the said Place, and the Equinoctial; and therefore may be either North or South.

Longitude of a Place, is an Arch of the Equinoctial, contain'd between the first Meridian, and the Meridian of the Place; and is either East or West.

NOTE, The first Meridian, is any Meridian made Choice of to reckon Longitude from; but our *English* Geographers of late, generally reckon their Longitude from the Meridian of *London*.

Di-

Fig. 1

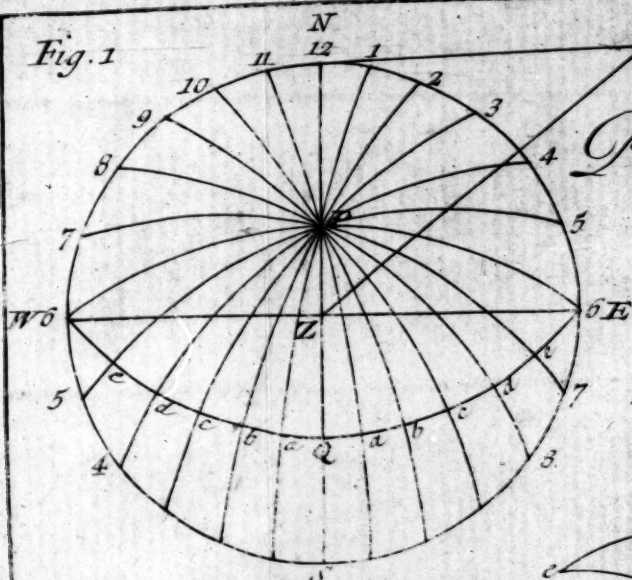


Plate 8

Fig. 2

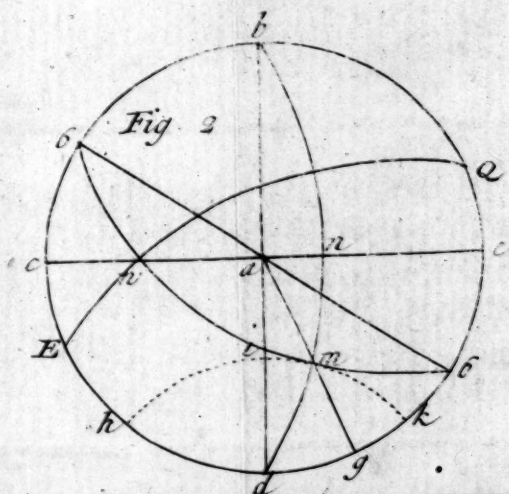


Fig. 3

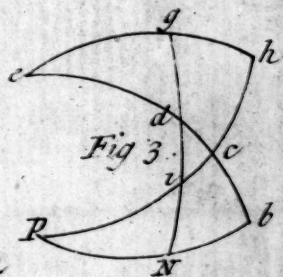
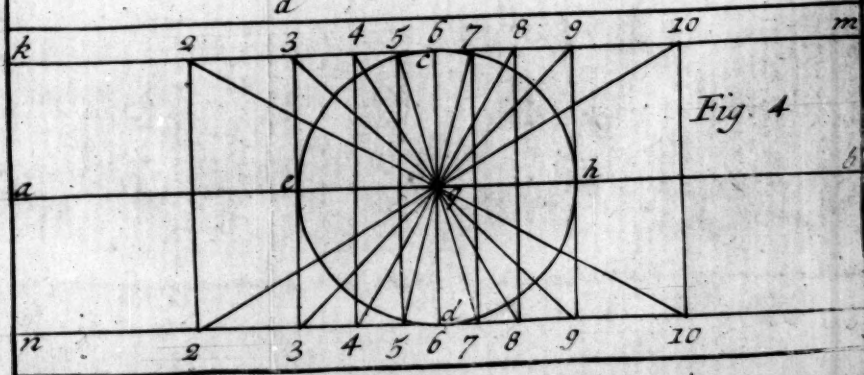


Fig. 4



Distance between two Places, is twofold, *viz.* in a Rumb, or in the Arch of a great Circle; but the latter being the nearest, and truest, is what I shall give some general Directions for at this Time.

Finding the Distance between any two Places, is commonly express'd under three Varieties, *viz.* 1. When Places differ only in Latitude. 2. When they differ only in Longitude. 3. When they differ both in Longitude and Latitude; but I shall reduce all these Varieties to this one

GENERAL RULE.

Having the Latitude and Difference of Longitude, of two Places given, find the Points representing them in the Stereographic Projection; then through the said Points draw a great Circle by Prop. 5. of Spherical Geometry; that Arch (contain'd between the said Points) measur'd by Prob. 8. of Spherical Geometry, gives the Distance requir'd, allowing every Minute a Mile, and every Degree 60 Miles.

E X A M P L E.

Plate 10.

Fig. 4.

Suppose two Places, both in Latitude 40 Degrees North, Difference of Longitude 38 Deg.

P.C.Q., then is *P* the North, *S* the South Pole, *E n Q* the Equinoctial; draw the Parallel of 40 Degrees *abc*, assume any Point therein, as *d*, for one of the two Places; through *d* draw the Meridian *P d S*, and observe where it cuts the Equinoctial, as in *o*; from *o* set off

the Difference of Longitude, 38 Degrees, to q , and draw the Meridian $P q S$, it will cut the Parallel $a b c$; in e , through d and e , draw the great Circle $R d e T$ the Arch $d e$ measur'd by Prob. 8. of spherical Geometry, gives the Distance requir'd.

CALCULATION.

The two Latitudes being equal, you have an Isoscelis Triangle $P d e$, in which is given $P e$, equal to $P d$, 50 Degrees, the Complement of Latitude, and the Angle $d P e$, 38 Degrees, the Difference of Longitude, to find the Arch $d e$; but in order to Calculation, let fall the Perpendicular $P w$, then in the right-angled Triangle $P w e$, right-angled at w , you have given $P e$ the Complement of Latitude 50 Degrees, and the Angle $w P e$, 19, half the Difference of Longitude, to find $w e$ half the Distance in the Arch, which is found by the following Proportion, Case 2.

As Radius,

To Sine Complement of Latitude $P e$.

So Sine $\frac{1}{2}$ Difference of Longitude in Minutes, $w P e$.

To Sine $\frac{1}{2}$ the Distance in Miles. $w e$ which doubled is the Distance requir'd.

For the Distance in the Parallel the Proportion, is

As Radius,

To Sine Comp. Latitude.

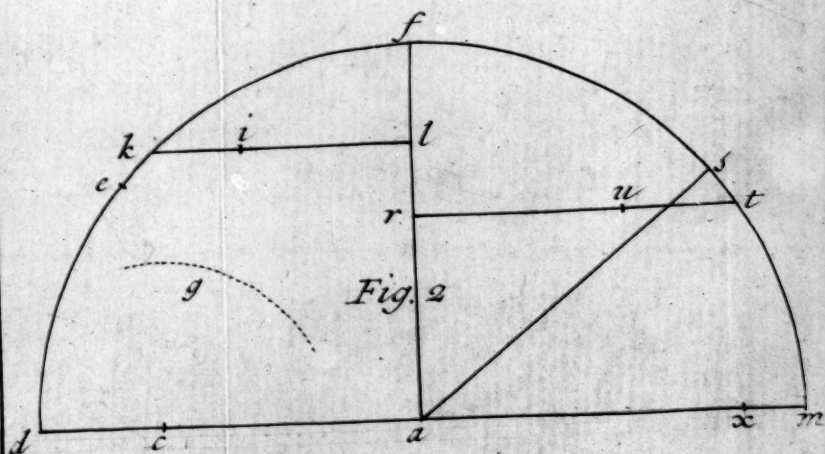
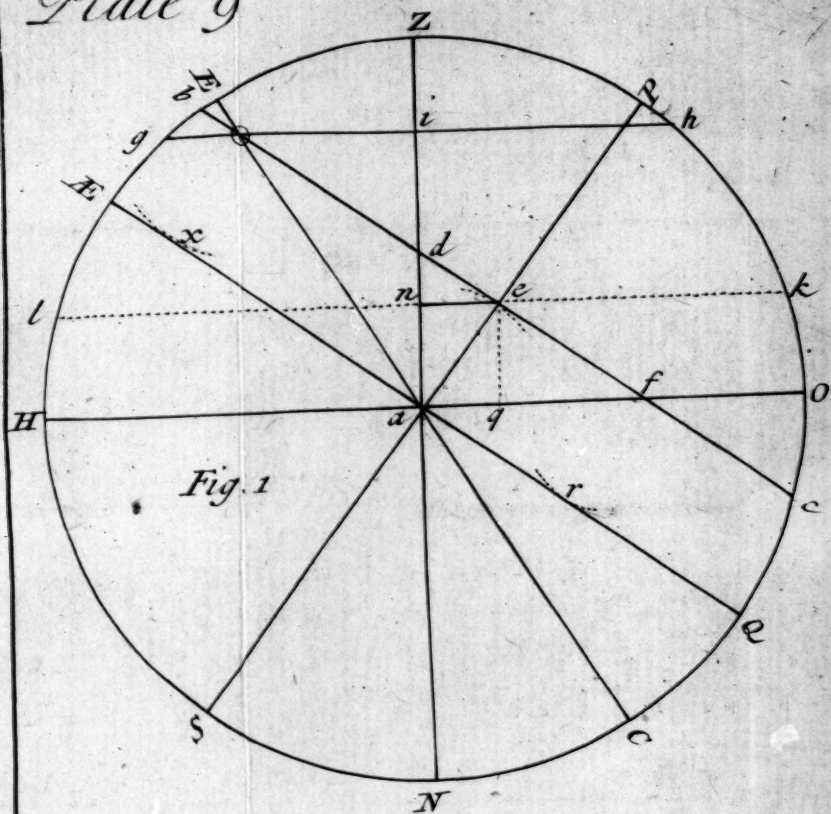
So Diff. Longitude in Minutes,

To the Distance in Miles in the Parallel,

Always more than the Distance in the Arch.

E X A M-

Plate 9



EXAMPLE 2.

If the Places differ both in Longitude and Latitude, as if one Place be in Latitude 40 North, and the other in Latitude 30 South, and differ in Longitude 58 Degrees; I desire to know their Distance.

P C Q, draw *a b c*, the Parallel of 40 North, and *x h z*, the Parallel of 30 South; assume a Point as in *d*, for one Place, and draw the Meridian *P d s*, and from the Place where it cuts the Equinoctial, as in *o*, set off the Difference of Longitude 58, from *o* to *y*, and draw the Meridian *P y S*, and observe where it cuts the Parallel *x h z*, which is in *h*, which represents the other Place; then through *d* and *h*, draw the Circle *i d h k*; the Arch *d h* is the Distance requir'd.

CALCULATION.

In the oblique Triangle *d P h*, you have given *d P*, the Complement of one Latitude 50 Degrees, and *P h* the other Latitude added to 90, viz. 120 Degrees, and the Angle at *P* 58 Degrees; the Difference of Longitude to find *d h* by Case the Eighth, of oblique spherical Triangles.

And thus by a right Understanding, and true Application of spherical Triangles, all the Cases in Geography, with respect to Distance of Places, may be solved; which is so plain, that I need not enlarge upon it.

I might have added something with respect to great Circle Sailing; but I shall not meddle with any Part of Navigation in this Treatise; because I shall (God willing) shortly publish a Book of Navigation, and the Use of the new Globular Projection of Sea Charts, in which great Circle Sailing is render'd easy and practicable, although never accounted so upon any other Projection; that impartial thinking Men may be convinc'd, Whether a fictitious cylindrical Projection, grounded upon a Supposition known to be false, or a true Globular Projection, grounded upon, and deduc'd from found and real Principles, be most applicable to Practice, and fittest to be depended upon for Truth and Certainty.

25 9062

APPEN-

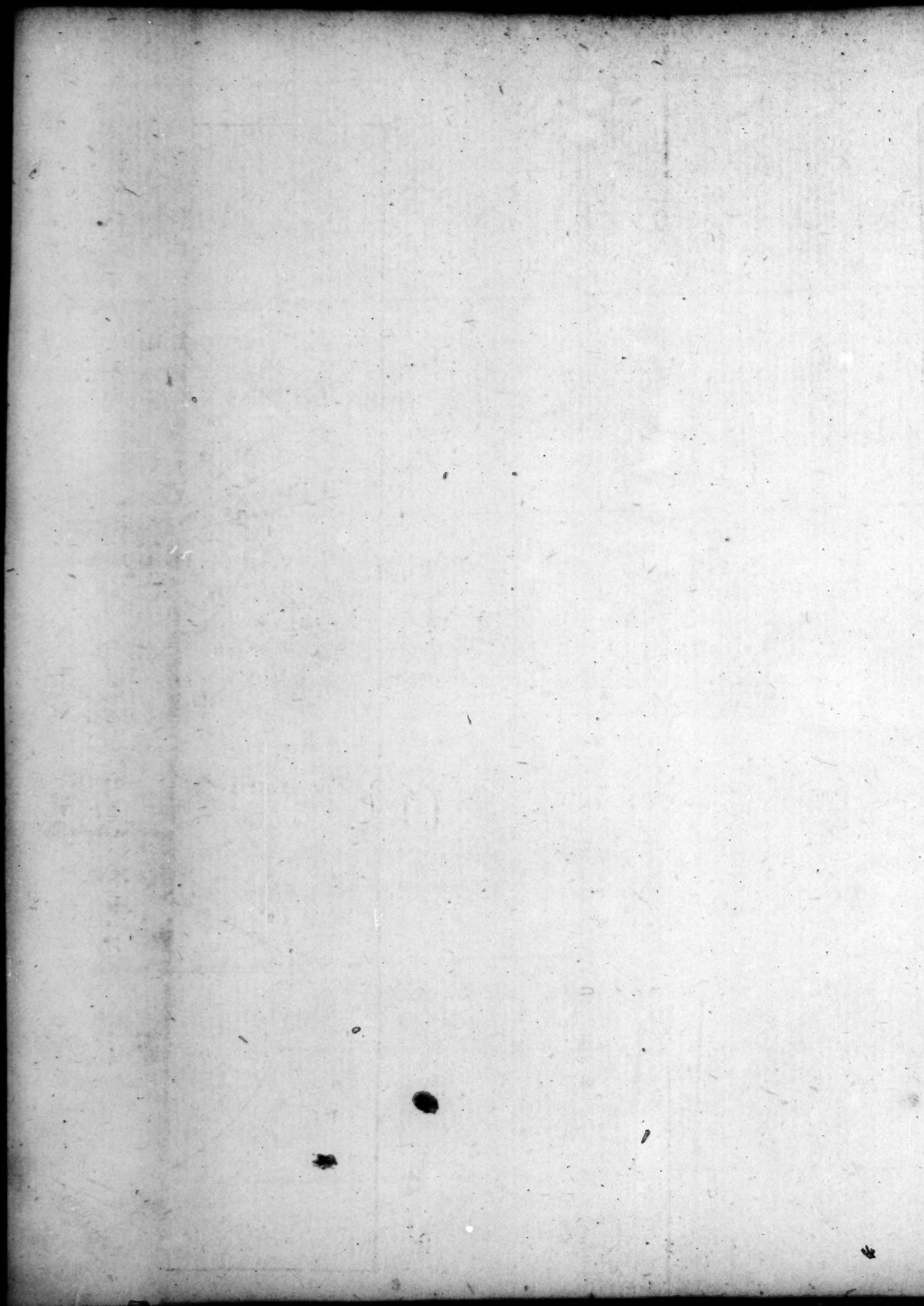


Plate 10.

Fig. 1.

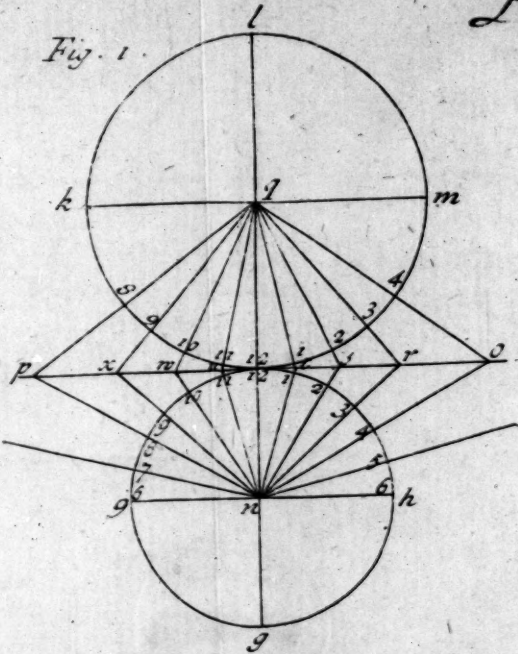


Fig. 2.

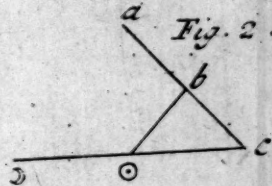


Fig. 3.

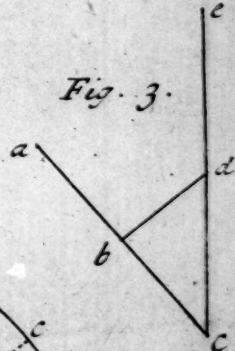
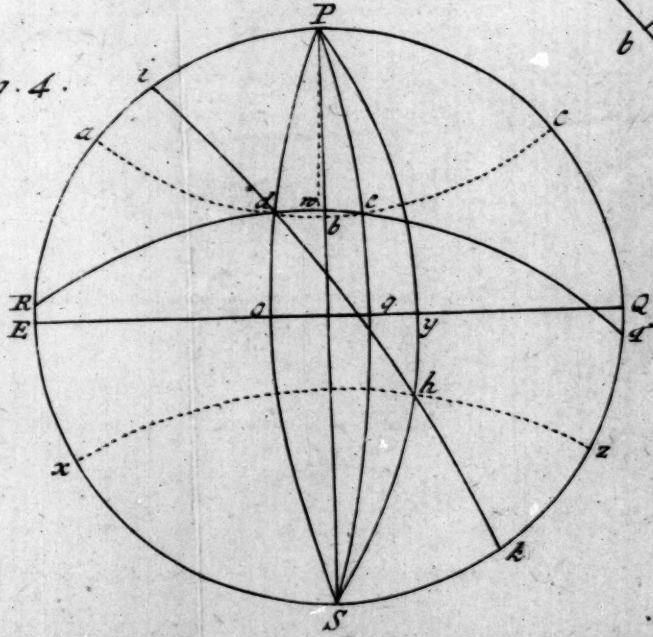


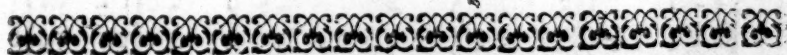
Fig. 4.





A P P E N D I X.

Containing Tables of Right Ascension, and of Oblique Ascension, and Descension ; for finding the Rising and Setting of the Sun, with the Rising, Southing, and Setting of the Moon, and the Southing of fixed Stars.



A Table of Right Ascension.

	♈	♉	♊	♋	♌	♍
D. H. M.	H. M. H. M.	H. M. H. M.	H. M. H. M.	H. M. H. M.	H. M. H. M.	H. M. H. M.
00	01 52 3	51 6	08	9 10 8		
10	41 55 3	55 6	48	12 10 12		
20	71 59 4	06	98	17 10 16		
30	112 34	46	138	21 10 20		
40	152 74	86	178	25 10 24		
50	182 124	126	228	29 10 27		
60	222 154	166	268	34 10 31		
70	262 194	216	318	38 10 35		
80	292 224	256	358	42 10 39		
90	332 264	296	398	46 10 42		
100	372 304	336	448	50 10 46		
110	402 344	386	488	54 10 50		
120	442 384	426	528	58 10 54		
130	481 424	466	579	2 10 57		
140	512 464	517	19	6 11 1		
150	552 504	557	59	10 11 5		
160	592 544	597	99	14 11 9		
171	32 585	37	149	18 11 12		
181	63 25	87	189	22 11 16		
191	103 65	127	229	26 11 20		
201	143 105	167	279	30 11 23		
211	183 145	217	319	34 11 27		
221	213 185	257	359	38 11 31		
231	253 225	297	399	41 11 34		
241	293 265	347	449	45 11 38		
251	333 315	387	489	49 11 42		
261	363 355	437	529	53 11 45		
271	403 395	477	569	57 11 49		
281	443 435	518	010	1 11 53		
291	483 475	568	510	5 11 56		
301	523 516	08	910	8 12 0		

A Table of Right Ascension.

♈			♉			♊			♋			♌		
D.	H.	M.	D.	H.	M.	D.	H.	M.	D.	H.	M.	D.	H.	M.
0	12	0	13	52	15	51	18	0	20	9	22	8		
1	12	4	13	55	15	55	18	4	20	13	22	12		
2	12	7	13	59	16	0	18	9	20	17	22	16		
3	12	11	14	3	16	4	18	13	20	21	22	20		
4	12	15	14	7	16	8	18	17	20	25	22	24		
5	12	18	14	11	16	12	18	22	20	29	22	27		
6	12	22	14	15	16	16	18	26	20	34	22	31		
7	12	26	14	19	16	21	18	31	20	38	22	35		
8	12	29	14	22	16	25	18	35	20	42	22	39		
9	12	33	14	26	16	29	18	39	20	46	22	42		
10	12	37	14	30	16	33	18	44	20	50	22	46		
11	12	40	14	34	16	38	18	48	20	54	22	50		
12	12	44	14	38	16	42	18	52	20	58	22	54		
13	12	48	14	42	16	46	18	57	21	2	22	57		
14	12	51	14	46	16	51	19	1	21	6	23	1		
15	12	55	14	50	16	55	19	5	21	10	23	5		
16	12	59	14	54	16	59	19	9	21	14	23	9		
17	13	3	14	58	17	3	19	13	21	18	23	12		
18	13	6	15	2	17	8	19	18	21	22	23	16		
19	13	10	15	6	17	12	19	22	21	26	23	20		
20	13	14	15	10	17	16	19	27	21	30	23	23		
21	13	18	15	14	17	21	19	31	21	34	23	27		
22	13	21	15	18	17	25	19	35	21	38	23	31		
23	13	25	15	22	17	29	19	39	21	41	23	34		
24	13	29	15	26	17	34	19	44	21	45	23	38		
25	13	33	15	31	17	38	19	48	21	49	23	42		
26	13	36	15	35	17	43	19	52	21	53	23	45		
27	13	40	15	39	17	47	19	56	21	57	23	49		
28	13	44	15	43	17	51	20	0	22	1	23	53		
29	13	48	15	47	17	56	20	5	22	5	23	56		
30	13	51	15	51	18	0	20	9	22	8	24	0		

A Table of Oblique Ascension.

D.	γ H. M.	δ H. M.	Π H. M.	$\ominus$ H. M.	Ω H. M.	☿ H. M.
0	18 0	18 52	20 0	21 47	0 18 3	9
1	18 2	18 54	20 3	21 51	0 23 3	14
2	18 3	18 56	20 6	21 56	0 29 3	20
3	18 5	18 58	20 9	22 1	0 35 3	26
4	18 7	19 0	20 12	22 5	0 40 3	32
5	18 9	19 2	20 15	22 9	0 46 3	38
6	18 10	19 4	20 18	22 14	0 51 3	43
7	18 12	19 6	20 21	22 19	0 57 3	49
8	18 13	19 8	20 24	22 24	1 2 3	55
9	18 15	19 10	20 27	22 28	1 8 4	1
10	18 16	19 13	20 30	22 33	1 14 4	7
11	18 18	19 15	20 34	22 39	1 20 4	12
12	18 20	19 17	20 37	22 43	1 26 4	18
13	18 22	19 19	20 41	22 48	1 32 4	23
14	18 23	19 21	20 44	22 53	1 37 4	29
15	18 25	19 23	20 48	22 58	1 43 4	34
16	18 27	19 26	20 52	23 3	1 49 4	40
17	18 29	19 28	20 55	23 9	1 54 4	46
18	18 31	19 30	20 59	23 14	2 0 4	52
19	18 32	19 32	21 2	23 19	2 6 4	58
20	18 34	19 35	21 6	23 24	2 12 5	4
21	18 36	19 37	21 10	23 29	2 18 5	9
22	18 37	19 39	21 14	23 34	2 23 5	15
23	18 39	19 42	21 18	23 40	2 29 5	21
24	18 41	19 45	21 22	23 45	2 34 5	26
25	18 43	19 48	21 26	23 51	2 40 5	32
26	18 45	19 50	21 30	23 56	2 46 5	37
27	18 47	19 53	21 34	0 2	2 52 5	43
28	18 48	19 55	21 39	0 7	2 58 5	48
29	18 50	19 58	21 43	0 13	3 3 5	54

A Table of Oblique Ascension.

♈			♍		♊		♏		♋		♉	
D.	H.	M.	H.	M.	H.	M.	H.	M.	H.	M.	H.	M.
0	6	0	8	51	11	43	14	13	16	0	17	8
1	6	5	8	57	11	48	14	17	16	3	17	10
2	6	11	9	2	11	53	14	21	16	6	17	12
3	6	17	9	8	11	58	14	25	16	8	17	14
4	6	23	9	14	12	4	14	30	16	10	17	15
5	6	28	9	20	12	9	14	34	16	13	17	17
6	6	34	9	26	12	15	14	38	16	15	17	19
7	6	40	9	32	12	20	14	42	16	18	17	21
8	6	46	9	37	12	25	14	46	16	21	17	22
9	6	51	9	43	12	30	14	50	16	23	17	24
10	6	57	9	48	12	36	14	54	16	25	17	26
11	7	3	9	54	12	41	14	58	16	28	17	28
12	7	8	10	0	12	46	15	1	16	30	17	29
13	7	14	10	6	12	51	15	5	16	32	17	31
14	7	20	10	11	12	56	15	9	16	35	17	33
15	7	26	10	17	13	2	15	12	16	37	17	34
16	7	31	10	23	13	7	15	16	16	39	17	36
17	7	37	10	28	13	12	15	19	16	41	17	38
18	7	43	10	34	13	17	15	22	16	43	17	40
19	7	48	10	40	13	22	15	26	16	45	17	42
20	7	54	10	46	13	27	15	30	16	48	17	44
21	7	59	10	51	13	32	15	33	16	50	17	45
22	8	6	10	57	13	36	15	36	16	52	17	47
23	8	11	11	3	13	41	15	39	16	54	17	49
24	8	17	11	9	13	46	15	42	16	56	17	50
25	8	22	11	14	13	51	15	45	16	58	17	52
26	8	28	11	20	13	55	15	48	17	0	17	53
27	8	34	11	26	13	59	15	51	17	2	17	55
28	8	40	11	31	14	4	15	54	17	4	17	57
29	8	46	11	37	14	9	15	57	17	6	17	59

A Table of Oblique Descension.

D.		H.		M.		H.		M.		H.		M.		H.		M.	
0	6	0	8	51	11	43	14	13	16	0	17	8					
1	6	5	8	57	11	48	14	17	16	3	17	10					
2	6	11	9	2	11	53	14	21	16	6	17	12					
3	6	17	9	8	11	58	14	25	16	8	17	14					
4	6	23	9	14	12	4	14	30	16	10	17	15					
5	6	28	9	20	12	9	14	34	16	13	17	17					
6	6	34	9	26	12	15	14	38	16	15	17	19					
7	6	40	9	32	12	20	14	42	16	18	17	21					
8	6	46	9	37	12	25	14	46	16	21	17	22					
9	6	51	9	43	12	30	14	50	16	23	17	24					
10	6	57	9	48	12	36	14	54	16	25	17	26					
11	7	3	9	54	12	41	14	58	16	28	17	28					
12	7	8	10	0	12	46	15	1	16	30	17	29					
13	7	14	10	6	12	51	15	5	16	32	17	31					
14	7	20	10	11	12	56	15	9	16	35	17	33					
15	7	26	10	17	13	2	15	12	16	37	17	34					
16	7	31	10	23	13	7	15	16	16	39	17	36					
17	7	37	10	28	13	12	15	19	16	41	17	38					
18	7	43	10	34	13	17	15	22	16	43	17	40					
19	7	48	10	40	13	22	15	26	16	45	17	42					
20	7	54	10	46	13	27	15	30	16	48	17	44					
21	7	59	10	51	13	32	15	33	16	50	17	45					
22	8	6	10	57	13	36	15	36	16	52	17	47					
23	8	11	11	3	13	41	15	39	16	54	17	49					
24	8	17	11	9	13	46	15	42	16	56	17	50					
25	8	22	11	14	13	51	15	45	16	58	17	52					
26	8	28	11	20	13	55	15	48	17	0	17	53					
27	8	34	11	26	13	59	15	51	17	2	17	55					
28	8	40	11	31	14	4	15	54	17	4	17	57					
29	8	46	11	37	14	9	15	57	17	6	17	59					

A Table of Oblique Descension.

D.	^h H. M.	^m H. M.	[†] H. M.	^v H. M.	[≡] H. M.	[✕] H. M.
0	18 0	18 52	20 0	21 47	0 18	3 9
1	18 2	18 54	20 3	21 51	0 23	3 14
2	18 3	18 56	20 6	21 56	0 29	3 20
3	18 5	18 58	20 9	22 1	0 35	3 26
4	18 7	19 0	20 12	22 5	0 40	3 32
5	18 9	19 2	20 15	22 9	0 46	3 38
6	18 10	19 4	20 18	22 14	0 51	3 43
7	18 12	19 6	20 21	22 19	0 57	3 49
8	18 13	19 8	20 24	22 24	1 2	3 55
9	18 15	19 10	20 27	22 28	1 8	4 1
10	18 16	19 13	20 30	22 33	1 14	4 7
11	18 18	19 15	20 34	22 39	1 20	4 12
12	18 20	19 17	20 37	22 43	1 26	4 18
13	18 22	19 19	20 41	22 48	1 32	4 23
14	18 23	19 21	20 44	22 53	1 37	4 29
15	18 25	19 23	20 48	22 58	1 43	4 34
16	18 27	19 26	20 52	23 3	1 49	4 40
17	18 29	19 28	20 55	23 9	1 54	4 46
18	18 31	19 30	20 59	23 14	2 0	4 52
19	18 32	19 32	21 2	23 19	2 6	4 58
20	18 34	19 35	21 6	23 24	2 12	5 4
21	18 36	19 37	21 10	23 29	2 18	5 9
22	18 37	19 39	21 14	23 34	2 23	5 15
23	18 39	19 42	21 18	23 40	2 29	5 21
24	18 41	19 45	21 22	23 45	2 34	5 26
25	18 43	19 48	21 26	23 51	2 40	5 32
26	18 45	19 50	21 30	23 56	2 46	5 37
27	18 47	19 53	21 34	24 2	2 52	5 43
28	18 48	19 55	21 39	24 7	2 58	5 48
29	18 50	19 58	21 43	24 13	3 3	5 54

A Table of the Moon's Motion.											Δ Ag.	Δ So. H.M.	
Hours.	12		13		14		15		16		A	So.	
	Deg.	d. m.	Deg.	d. m.	Deg.	d. m.	Deg.	d. m.	Deg.	d. m.			
1	0	30	0	32	0	35	0	37	0	40	7	5	36
2	1	0	1	5	1	10	1	15	1	20	8	6	24
3	1	30	1	37	1	45	1	52	2	0	9	7	12
4	2	0	2	10	2	20	2	30	2	40	10	8	0
5	2	30	2	43	2	35	3	7	3	20	11	8	48
6	3	0	3	15	3	30	3	45	4	0	12	9	36
7	3	30	3	48	4	5	4	22	4	40	13	10	24
8	4	0	4	20	4	40	5	0	5	20	14	11	12
9	4	30	4	53	5	15	5	37	6	0	15	12	0
10	5	0	5	25	5	50	6	15	6	40	16	0	M 48
11	5	30	5	58	6	25	6	52	7	20	17	1	36
12	6	0	6	30	7	0	7	30	8	0	18	2	24
13	6	30	7	3	7	35	8	7	8	40	19	3	12
14	7	0	7	35	8	10	8	45	9	20	20	4	0
15	7	30	8	8	8	45	9	22	10	0	21	4	48
16	8	0	8	40	9	20	10	0	10	40	22	5	36
17	8	30	9	13	9	55	10	37	11	20	23	6	24
18	9	0	9	45	10	30	11	15	12	0	24	7	12
19	9	30	10	18	11	5	11	52	12	40	25	8	0
20	10	0	10	50	11	40	12	30	13	20	26	8	48
21	10	30	11	23	12	15	13	7	14	0	27	9	36
22	11	0	11	55	12	50	13	45	14	40	28	10	24
23	11	30	12	28	13	25	14	22	15	20	29	11	12
24	12	0	13	0	14	0	15	0	16	0	30	12	0

THERE is in all Cases two Numbers to be subtracted from each other, the Remainder is the Answer to the Question, in Hours and Minutes: Of these two Numbers the Sun's right Ascension is always one, because we begin to reckon the Astronomical Hours at Noon; then for Rising, use oblique Ascension; for Southing, use right Ascension; and for Setting, use oblique Descension, viz. If you seek for the Sun's Rising, subtract his right Ascension from his oblique Ascension, the Remainder is the Time of Sun's Rising; but if you seek the Time of Sun's Setting, subtract his right Ascension, from his oblique Descension, the Remainder is the Time of Sun's Setting. Again, If you desire to know the Time of the Moon's Coming to South, subtract the Sun's right Ascension from the Moon's right Ascension, the Remainder is the Time of Moon's Southing; but if you would know the Time of the Moon's Setting, subtract the Sun's right Ascension, from the Moon's oblique Descension, the Remainder is the Time of Moon's Setting, borrowing 24 Hours if Subtraction cannot otherwise be made, and if the Remainder, be more than 12, cast away 12, and the Remainder is the Hour and Minute, in the Morning; but if less than 12, it is the Hour and Minute in the Afternoon.

If you seek the Rising, Southing, or Setting of the Moon, and would be exact; you must first compute in your Mind about what Time it will happen, and adjust her Place to that Hour, and then work as before.

EX-

190 *The Use of the TABLES.*

E X A M P L E.

I desire to know the Time of Sun's Rising, *Octob.* 19. 1720. I look in the *Ephemeris*, and find the Sun is in 7 D. 9. M. of $\mathfrak{m}$. I look for 7 D. (rejecting the Minutes) and find the oblique Ascension of 7 D. of $\mathfrak{m}$, is 9 H. 9 32 32 M. and the right Ascension thereof is 14 H. 19 M. which subtracted (having first suppos'd 24 Hours to be added to the oblique Ascension, as before directed) the Remainder is 19 13 12 13; from which abating 12, the Remainder 7 H. 13 M. is the Time of Sun's Rising, *October* 19, as was requir'd.

E X A M P L E 2.

I desire to know the Time of Moon's Setting, *Nov.* 21. 1720.

If we take the Place of the Moon at Noon, and use it as her true Place, it would beget a small Error, but for more Exactness, consider that the Moon is 3 Days old, and therefore will not set before 6 or 7 a Clock; I therefore compute her Place for 6 a Clock, thus;

I find the Moon, *Nov.* 21, at Noon, is $\mathfrak{w}$ 16 D. 55 M. her Motion from that Day at Noon, till *Nov.* 22. at Noon, is about 15 Degrees; I therefore look in the fourth Column of the last Table, under 15 Degrees, and against 6 Hours, I find 3 D. 45 M. which is the Degrees and Minutes the Moon has gone since Noon, that added to her Place at Noon, $\mathfrak{w}$ 16 55, makes $\mathfrak{w}$ 20 40, which being near 21 Degrees, and not regarding odd Minutes in this Case, I seek the oblique Descension of $\mathfrak{w}$ 21, and find it is 23 D. 29 M. I also

The Use of the TABLES. 191

I also find the right Ascension of the Sun (who that Day enters 11 Degrees of φ) the right Ascension of 11 of φ , is 16 38, which subtracted from 23 29, the Remainder 6 51, viz. 6 Hours 51 Minutes, is the Time of Moon's Setting, Nov. 21, in the Afternoon.

E X A M P L E 3.

It is requir'd to find the Time of the Moon's Southing, June 2. 1720.

First, Consider it is Quarter-Day of the Moon, and therefore the Moon Souths sometime near 6 a Clock, then to compute her Place from the Ephemeris; her Place June 2, at Noon, is in M 19 10; and on the third Day, is M 2 51; the Difference, or daily Motion of the Moon, is 13 41, which being more than 13 30, I call 14 Degrees, under which I look in the Table of the Moon's Motion, and against 6 Hours (the Time that I calculate for) I find that in 6 Hours the Moon is remov'd 3 Deg. 30 Min. which added to her Place at Noon, M 19 10, gives M 22 40, which being above half a Degree more than 22, I call 23, and proceed as below.

	d.	m.
Moon's Place M 23 right Ascension, —	11	34
Sun's Place II 23 right Ascension, —	5	29

Differ. is the Time of Moon's Southing. 6. 5
But

192 *The Use of the TABLES.*

But lest it should be thought difficult to give a general Guess of the Time of the Moon's Southing, by her Age; I have annex'd a Table to the Table of the Moon's Motion, in which find the Moon's Age in the Column, intitl'd [*D's Age*] and against it in the next Column, you have the Moon's Southing.

EXAMPLE 4.

Suppose the Moon is 11 Days old; I find 11 in the Column of *D's Age*, and against it is 8 48, Afternoon for the Moon's Southing, which might serve indeed in common Cases, and is in any Case exact enough to give a near Estimate of the Time of her Southing, in order to adjust her Place thereto, as before directed; for although this Table only divides 24 Hours, equally among 30 Days, without regard to the Inequality of the Moon's Motion; yet by the aforementioned Method, all Accidents are accounted for, except some little Alteration that the Moon's Rising, Southing, and Setting, are liable to, occasion'd by her Latitude, a Thing inconsiderable in most Cases, and too nice for our intended Brevity, to enter into a Calculation of.

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By the same Method, you may find the Southing of any fix'd Star; but in Stars that have great Latitude from the Ecliptick, their Rising and Setting cannot be found exactly; because their Distance from the Ecliptick makes them rise or set, long before or after that Point of the Ecliptick, which their Circle of Longitude Intersects.

F I N I S.

